Spread Histogram – A Method for Calculating Spatial Relations Between Objects

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Summary. This paper presents a novel approach called Spread Histogram for calculation of spatial relations between objects. It allows to determine such relations as *INSIDE*, *OUTSIDE*, *ENCOMPASS*. Additionally, the method cooperates very well with standard histogram methods like Histogram of Angles for determining the directional spatial relations.

1 Introduction

The need of spatial relations determination is very high in research areas such as image understanding. It is used for the image content description and is one of the most important sources of knowledge about image content. There are many approaches used for spatial relations determination [1][2][3][4].

Histogram of angles, a method for extracting spatial relation between two objects is able to represent *RIGHT OF*, *LEFT OF*, *BELOW*, *ABOVE* relations [3]. It takes in the account an angle between vector connecting two points, one from the first object and second from the other, and one of the axes. Such angle is calculated for every pair of points. Sets of angles are labeled by proper spatial relations. The dominant relations between the objects can be read from the histogram. Force Histogram [2][5], also called F–Histogram, uses the value of scalar of forces associated with certain angles between two objects. R–Histogram [3][6], the extension of Histogram of Angles method, uses the concept of labeled distance to handle *INSIDE* and *OUTSIDE* spatial relations. Labeled distance calculation is partially based on the Euclidian distance calculation. Euler Histogram [4][7][8] and its extensions are based on division of image into cells and assigning a bucket to each cell. Values of buckets are modified based on presence of objects in relevant cells.

This paper presents a novel approach called Spread Histogram for representing spatial relations between two objects. It allows, with cooperation of Histogram of Angles, to determine the relations *RIGHT OF*, *LEFT OF*, *ABOVE*, *BELOW*, *INSIDE*, *OUTSIDE*, and, what seems to be important, the

ENCOMPASS relation. Results are presented for exemplary pairs of objects. The last part of the paper contains a short discussion of the S–Histogram method and further research directions.

2 The method

Let us assume X and Y to be sets of points representing two shapes:

$$X = \{x_1, x_2, ..., x_m\} \quad (1)$$

$$Y = \{y_1, y_2, ..., y_n\} \quad (2)$$

For every x_i there can be calculated a vector of angles between x_i and every point in Y:

$$A_i = [\alpha_1, \alpha_2, ..., \alpha_n] \quad (3)$$

Let us order the angles in the vector of angles A_i in an ascending manner:

$$\forall_{a \in \{1,2,...,n-1\}} \forall_{b \in \{a+1,...,n\}} \quad \alpha_a \leq \alpha_b \quad (4)$$

Let us calculate the *coefficient of spread* for the given point x_i:

$$\beta_i = max(\alpha_2 - \alpha_1, \alpha_3 - \alpha_2, ..., \alpha_n - \alpha_{n-1}, 2\pi - \alpha_n + \alpha_1) \quad (5)$$

If (x_i is inside Y and x_i is not on the border of Y) or x_i is encompassed by Y then

$$(n \to \infty) \Rightarrow (\beta_i \to 0) \quad (6)$$

If (x_i is not inside Y or x_i is on the border of Y) and x_i is not encompassed by Y then

$$(n \to \infty) \Rightarrow (\beta_i \gg 0) \quad (7)$$

The difference between *INSIDE*, *ENCOMPASS* and a partial *ENCOMPASS* has been shown in fig. 1.

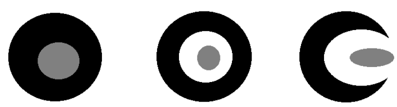


Fig. 1. Examples of *INSIDE*, *ENCOMPASS* and partial *ENCOMPASS* spatial relations

The histogram of β angles is calculated for every $x \in X$. Based on this statistic, the spatial relations between X and Y can be determined. Calculation of β value require angle sorting. Standard sorting algorithms take $O(n \log n)$, but in this case bucket sorting ($O(n)$) is enough to calculate β value with a given precision. Let us denote $S'(X, Y, \beta)$ as an unnormalized Spread Histogram (eq. 8).

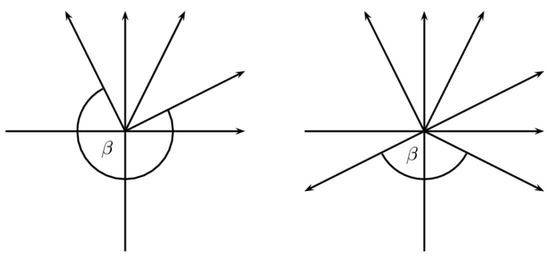
$$\forall_{i \in \{1,2,\ldots,m-1\}} \quad S'(X, Y, \beta) = \begin{cases} S'(X, Y, \beta) + 1 & if \quad \beta_i = \beta \\ S'(X, Y, \beta) & otherwise \end{cases} \tag{8}$$

After the calculation of histogram it is required to normalize it. Let us denote $S(X, Y, \beta)$ as a normalized Spread Histogram (eq. 9).

$$S(X, Y, \beta) = \frac{S'(X, Y, \beta)}{\sum_{\beta \in <0, 2\pi)} S'(X, Y, \beta)} \tag{9}$$

Spread Histogram (also called S–Histogram) is both rotation and scale invariant. Rotating and Scaling objects do not introduce any changes in the S–Histogram.

Spread Histogram can be very easily calculated when the Histogram of Angles is calculated. It takes $O(nm + mk)$ to calculate the histogram, where m is the number of points in X, n is the number of points in Y and k is the number of "bins" in bucket sorting used to calculate the β values. Exemplary vectors with calculated β angle have been shown in fig. 2.



(a) (b)

Fig. 2. Exemplary β angles, point outside object (a), point inside object (b)

Classification rules

The classification of spatial relation is based on a set of functions. These functions represent the intensity of each of the spatial relations. Values of every function are normalized. Let us denote $A(X, Y, \alpha)$ as a Histogram of Angles value for α angle.

$$Left(X, Y) = \frac{\sum_{\alpha \in <3/2\pi, 2\pi) \cup <0, \pi/2)} A(X, Y, \alpha)}{\sum_{\alpha \in <0, 2\pi)} A(X, Y, \alpha)} \tag{10}$$

$$Right(X,Y) = \frac{\sum_{\alpha \in <\pi/2,3/2\pi)} A(X,Y,\alpha)}{\sum_{\alpha \in <0,2\pi)} A(X,Y,\alpha)} \quad (11)$$

$$Below(X,Y) = \frac{\sum_{\alpha \in <0,\pi)} A(X,Y,\alpha)}{\sum_{\alpha \in <0,2\pi)} A(X,Y,\alpha)} \quad (12)$$

$$Above(X,Y) = \frac{\sum_{\alpha \in <\pi,2\pi)} A(X,Y,\alpha)}{\sum_{\alpha \in <0,2\pi)} A(X,Y,\alpha)} \quad (13)$$

$$Inside(X,Y) = \frac{\sum_{\beta \in <0,\pi)} S(X,Y,\beta)}{\sum_{\beta \in <0,2\pi)} S(X,Y,\beta)} \quad (14)$$

$$Outside(X,Y) = \frac{\sum_{\beta \in <\pi,2\pi)} S(X,Y,\beta)}{\sum_{\beta \in <0,2\pi)} S(X,Y,\beta)} \quad (15)$$

Equations 10, 11, 12 and 13 are based only on Histogram of Angles. Equations 14 and 15 are based on the Spread Histogram. Additionally the Spread Histogram allows to determine *ENCOMPASS* and *IS ENCOMPASSED* spatial relations. For both of these relations Histogram of Angles is almost identical. It is required to have additional information and such information is provided by Spread Histogram.

Let us define a function $f^A_{cont}(X,Y)$ based only on Histogram of Angles. Function $f^A_{cont}(X,Y)$ has normalized values and gives the information how much *X encompasses Y* or *Y encompasses X*. All experiments in this paper for *ENCOMPASS* and *IS ENCOMPASSED* have been performed using heuristic function f^A_{cont} given by eq. 17.

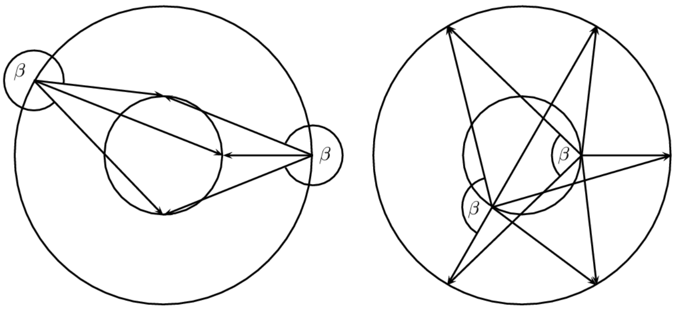
$$Encompass(X,Y) = f^A_{cont}(X,Y) \frac{\sum_{\beta \in <\pi,2\pi)} S(X,Y,\beta)}{\sum_{\beta \in <0,2\pi)} S(X,Y,\beta)} \quad (16)$$

$$f^A_{cont}(X,Y) = 16 Left(X,Y) Right(X,Y) Below(X,Y) Above(X,Y) \quad (17)$$

Another important spatial relation is *IS ENCOMPASSED*. This method is not able to differentiate *INSIDE* and *IS ENCOMPASSED*, but is able to differentiate *OUTSIDE* and *ENCOMPASS*. The differentiation between *INSIDE* and *IS ENCOMPASSED* can be very easy performed by calculating $A(Y,X,\alpha)$, $S(Y,X,\beta)$ and $Encompass(Y,X)$ as given by eq. 18.

$$IsEncompassed(X,Y) = Encompass(Y,X) \quad (18)$$

For an object placed in the center of *encompassed area* values of all directional functions are close to 0.5. To normalize for an object placed in the center the value of the function has to be multiplied by 16. S–Histogram enables differentiation between *ENCOMPASS* and *IS ENCOMPASSED*. For each point of object encompassing another object β angles have large values (fig. 3a). All vectors outgoing from one point have similar direction. For each point, the



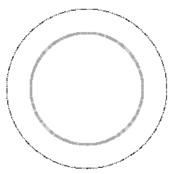
(a) (b)

Fig. 3. β angles for *ENCOMPASS* (a) and *IS ENCOMPASSED* (b)

directions of vectors are completely different, but the β angle still is large. For each point of object encompassed by another object β angles have small values (fig. 3b). All vectors outgoing from one point are distributed along the whole angle domain. Such situation occurs for every point in the object.

3 Results of experiments

All experiments have been conducted for spatial relations between two objects. The black–dotted object is referred as object X, the grey–crossed object is referred as object Y.



Relation	Result
X right of Y	0.50
X left of Y	0.49
X below Y	0.49
X above Y	0.50
X inside/enc. by Y	0.00
X outside Y	1.00
X encompass Y	0.95

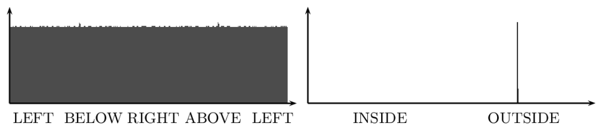
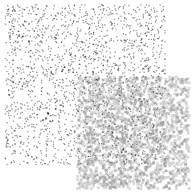


Fig. 4. Object X encompasses object Y

Figure 4 shows object X encompassing object Y. Spread Histogram has higher than 0 values for $\beta \in< \pi, 2\pi)$. Histogram of Angles shows uniform distribution of all angles. All directional spatial relations are present between these two objects.



Relation	Result
X right of Y	0.21
X left of Y	0.78
X below Y	0.21
X above Y	0.78
X inside/enc. by Y	0.29
X outside Y	0.70
X encompass Y	0.30

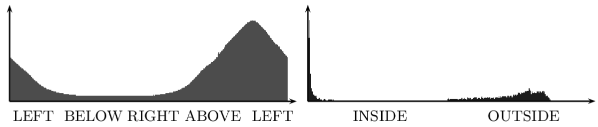
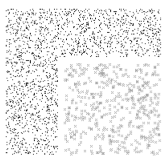


Fig. 5. Object X partially inside object Y

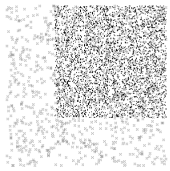
Figure 5 shows object X partially inside object Y. Spatial Histogram allows for approximate calculation how much of the object is *INSIDE* or *IS ENCOMPASSED* by another object. As shown in fig. 5, $Inside(X, Y) = 0.29$ of object X is inside object Y. Spread Histogram has greater than 0 values for both $\beta \in < 0, \pi)$ and $\beta \in < \pi, 2\pi)$. It is impossible to determine *INSIDE* and *OUTSIDE* spatial relations only with Histogram of Angles. Situation with very similar results from Histogram of Angles as in fig. 5 have been shown in fig. 6. It is required to use information from the Spread Histogram to calculate the difference.



Relation	Result
X right of Y	0.17
X left of Y	0.82
X below Y	0.18
X above Y	0.81
X inside/enc. by Y	0.00
X outside Y	1.00
X encompass Y	0.32

Fig. 6. Object X outside object Y, X partially encompasses Y

Figure 7 shows object X inside object Y. Spread Histogram has greater than 0 values only for $\beta \in < 0, \pi)$. For every point of X β angles are small, because points of Y are around it.



Relation	Result
X right of Y	0.65
X left of Y	0.34
X below Y	0.34
X above Y	0.65
X inside/enc. by Y	1.00
X outside Y	0.00
X encompass Y	0.00

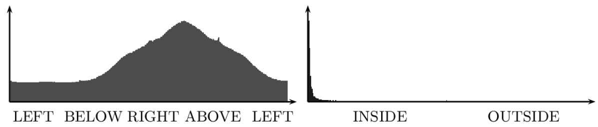
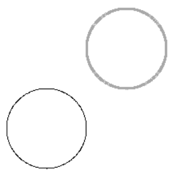


Fig. 7. Object X inside object Y

Figure 8 shows object X outside object Y. Spread Histogram has greater than 0 values only for $\beta \in < \pi, 2\pi)$. For every point of X β angles are large, because all points of Y are placed only in one direction from it.



Relation	Result
X right of Y	0.00
X left of Y	1.00
X below Y	1.00
X above Y	0.00
X inside/enc. by Y	0.00
X outside Y	1.00
X encompass Y	0.00

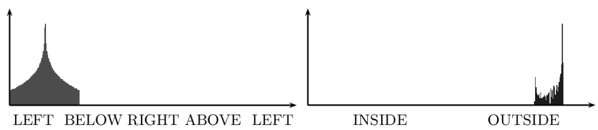


Fig. 8. Object X outside Y

Spread Histogram allows in certain cases to distinguish between *NEAR* and *FAR* spatial relations. It is working very well for rather circular objects, but gives false result for very long and flat objects. *FAR* spatial relation is present if Histogram of Angles has detected 1 or 2 of non–opposite directional spatial relations. Additionally Spread Histogram can have larger than 0 values for only very large β values. *NEAR* spatial relation is the opposite, it is present only if one *FAR* spatial relation conditions is not met.

4 Discussion and further research

Presented method allows in a very simple and precise way to distinguish between spatial relations like *INSIDE* and *OUTSIDE*. Additionally it presents a way to distinguish between *OUTSIDE*, *ENCOMPASS* and *IS ENCOMPASSED*, which is often needed in image analysis. In some degree it also handles *FAR* and *NEAR* spatial relations. The advantage of the Spread Histogram is that it is based only on angles calculation, which makes very easy to incorporate it into Histogram of Angles.

Further research will focus on the following aspects:

- Replacing functions given by eqs 10 to 18 with a classifier with supervised learning. This will allow to adjust the method to user preferences.
- Using S–Histogram method for determining spatial relations in real-life images. The method is plan to be used on skin cancer images for determining the relations between melanocytic lesions [9].

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