Homework 1

1. Consider the problem of adding “n” numbers. Assume that one person can add two numbers in time $t_c$. How long will a person take to add “n” numbers. Now assume that eight people are available for adding these “n” numbers and that it is possible to divide the list into eight parts. The eight people have their own pencils and paper (on which perform addition), are equally skilled, and can add two numbers in time $t_c$. Furthermore, A person can pass on the result of an addition (in form of a single number) to the person sitting next to him or her in time $t_c$. How long will it take to add n numbers in the following scenarios:
   (a) All eight people are sitting in a circle,
   (b) The eight person are sitting in two rows of four people each,
   (c) The eight person are sitting in the nodes of 3-dimensional hypercube network.

2. Consider again the problem of adding “n” numbers. Assume that one person takes time $t_c(n-1)$ to add these numbers. Is it possible for “p” people to add this list in time less than $t_c(n-1)/p$? Justify your answer.

3. Determine equations for diameter, bisection width, arc connectivity and cost (number of links) for the following static network topologies: Completely-connected, star, complete binary tree, linear array, ring, 2-D mesh without wraparound, 2-D wraparound mesh and hypercube.

4. How many distinct labelings exist for a d-dimensional hypercube. Justify your answer. A cycle in a graph is defined as a path originating and terminating at the same node. The length of the cycle is the number of edges in the cycle. Show that there are no odd-length cycles in a d-dimensional hypercube.

5. The label in a d-dimensional hypercube use d bits. Fixing any k of these bits, show that processors whose labels differs in the remaining d-k bit position form a (d-k)-dimensional subcube compose of $2^{(d-k)}$ processors.

6. Let A and B be two processors in a d-dimensional hypercube. Define $H(A, B)$ to be the Hamming distance between A and B, and $P(A, B)$ to be the number of distinct paths connecting A and B. These paths are called parallel paths and have no common processors other than A and B. Prove the following:
   (a) The minimum distance in terms of communication links between A and B is given by $H(A, B)$.
   (b) The total number of parallel paths between any two processors is $P(A, B) = d$.
   (c) The number of parallel paths between A and B of length $H(A, B)$ is $P_{\text{length}=H(A, B)}(A, B) = H(A, B)$
   (d) The length of the remaining $d - H(A, B)$ parallel paths is $H(A, B) + 2$. 
