Modeling spot, forward and option prices of several commodities: a regime switching approach

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Agenda

1. **Introduction**
   - Context
   - Proposition
2. **Econometric approach**
   - Introduction of the stochastic discount factor
   - Historical and Risk Neutral dynamics
   - Derivatives pricing
3. **Modeling choices**
   - Description
   - Calibration issues
   - Illustration on forward prices reconstruction
4. **Conclusion and perspectives**

Introduction

Introduction: commodity modeling in EDF

- Short term (1 day $\rightarrow$ 2 weeks) prediction
- Market comprehension
- Mid term risk management
  - Gross energy margin prediction
  - Risk measurement
  - Hedging
- Pricing
  - Valuation of Production assets
  - Valuation of flexibilities in supply contracts
- Investment decision

What is needed

- **Portfoltio exposition**
  - Power and fuels spot prices
  - Power and fuels forward products
- **What is needed**
  - forward products modeling
  - spot prices modeling
  - calibration procedures
  - for several commodities
- **Proposition: an econometric approach**
  - as an alternative to the classic continuous-time approach
  - capturing the main properties of spot prices (spikes, link beteen commodities)
  - defining the needed concepts to obtain a risk-neutral probability (the concept of the stochastic discount factor)

Econometric approach

The Stochastic Discount Factor and the Risk-Neutral probability

- **Fundamental assumptions:**
  1. Existence of prices
  2. Absence of arbitrage opportunity (static)
  3. Absence of arbitrage opportunity (dynamic)

- **Consequence [Bertholon et. al. 2008, Monfort & Féron, 2012]:**
  - Existence of a **stochastic discount factor** (SDF) $M_{t,T}$

$$\begin{aligned} M_{t,T} &> 0 \\ M_{t,T} &= M_{t,t+1} M_{t+1,t+2} \dots M_{T-1,T} \end{aligned}$$

  - Existence of a risk-neutral probability

$$\mathbb{E}^{\mathbb{Q}}\left[X_T | \boldsymbol{w}_t\right] = \mathbb{E}_t^{\mathbb{Q}}\left[X_T\right] = \mathbb{E}_t^{\mathbb{P}}\left[\frac{M_{t,T}}{\mathbb{E}_t\left[M_{t,T}\right]} X_T\right]$$

  - Pricing of a derivative whose payoff is $g(S_T)$:

$$p_t\left[g(S_T)\right] = \mathbb{E}_t^{\mathbb{Q}}\left[g(S_T) e^{-r(T-t)}\right]$$

Modeling point of view

- 3 elements
  - Historical dynamics ($\mathbb{P}$)
  - Stochastic Discount Factor ($M_{t,t+1}$)
  - Risk-Neutral dynamics ($\mathbb{Q}$)

  Two elements have to be defined, the third is deduced.

- Incomplete market : There is an infinity of SDF consistent with observed prices.

- Restriction (*a priori*) of the acceptable set of SDF with a affine exponential form to obtain closed-form formulas of pricing

$$M_{t,t+1} = exp\left\{\alpha_t(\boldsymbol{w}_t) + \beta_t(\boldsymbol{w}_t)\boldsymbol{w}_{t+1}\right\}$$

  with $\boldsymbol{w}_t$ the all set of random variables at date t.

- This restriction must, at least, respect AOA properties (i.e. some internal consistency conditions).

Our modeling choices

- Regime switching VAR(m) for the historical dynamics
  - with eventually non-homogeneous Markov Chain
  - Capturing stylized facts of the spot prices (spikes, changes of volatility...)
  - Using classic estimation procedures (Hamilton filter)

- Regime switching VAR(m) for the risk-Neutral dynamics
  - with homogeneous Markov Chain
  - Allowing closed-form (or quasi-closed form) formulas for forward prices and European options
  - Using classic estimation procedures (Kalman, extended-Kalman filters)

Modeling choice

Information

- Discrete time of the economy ($t = 1, 2, \dots, T$)
- Information at date t
  - Spot prices of N commodities, denoted by $(S_{1,t}, \dots, S_{N,t})'$
  - Convenience yields $(\delta_{1,t}, \dots, \delta_{N,t})'$ linking the forward price $F_n(t, t+1)$ to the spot price $S_{n,t}$ [Schwartz, 1997]

$$F_n(t, t+1) = S_{n,t} e^{r_t - \delta_{n,t}}$$

    Remark: for Power spot prices, there will be no associated convenience yield

  - The risk free rate r_t
  - Endogenous qualitative variable z_t, valued in $(e_1, \dots, e_K)$
- Whole set of information at dant t

$$\boldsymbol{w}_t = (S_{1,t}, \dots, S_{N,t}, \delta_{1,t}, \dots, \delta_{N,t}, r_t, z_t)'$$

Historical dynamics

- **Notations**
  - Residual spot price logarithm $\ln(S_{n,t}) = \nu_{n,t} + s_{n,t}$, $n = 1, \ldots, N$.
  - $\boldsymbol{x}_t = (s_{1,t}, \ldots, s_{N,t}, \delta_{1,t}, \ldots, \delta_{N,t}, r_t)'$

- **Switching regime VAR(1)** on the residual noises:

$$\boldsymbol{x}_{t+1} = \boldsymbol{M}_1 z_{t+1} + \boldsymbol{M}_0 z_t + \boldsymbol{\Phi} \boldsymbol{x}_t + \Sigma^{1/2}(z_{t+1}, z_t)\varepsilon_{t+1} \quad (1)$$

  ε_{t+1} being a standard Gaussian white noise process of size $2N + 1$, $\Sigma^{1/2}(z_{t+1}, z_t)$ a symmetric positive definite matrix function

- **Markov chain on** z_t

$$\mathbb{P}(z_{t+1} = e_j | z_t = e_i, s_t) = \pi_{ijt} \quad (2)$$

  z_t a state variable which takes K values corresponding to K regimes characterized by mean μ_i variance σ_i^2 (ex: Nominal regime, high volatility regimes, spikes regime)

Risk-neutral dynamics

Proposition 1

Assuming the historical dynamics described by (1) and (2),
if the stochastic discount factor has the following form:

$$M_{t,t+1} = \exp\left\{-r_t - \frac{1}{2}\alpha'(z_{t+1}, z_t, x_t)\alpha(z_{t+1}, z_t, x_t) + \alpha'(z_{t+1}, z_t, x_t)\varepsilon_{t+1} + \beta'(z_t, x_t)z_{t+1}\right\}$$

with:

$$\begin{aligned}
\beta(e_i, x_t) &= \left(\log\frac{\tilde{\pi}_{i1}}{\pi_{i1}(e_1|e_i, x_t)}, \dots, \log\frac{\tilde{\pi}_{iK}}{\pi_{iK}(e_K|e_i, x_t)}\right)' \\
\alpha(z_{t+1}, z_t, \boldsymbol{x}_t) &= \Sigma^{-1/2}(z_{t+1}, z_t)\left[(\tilde{\boldsymbol{\Phi}} - \boldsymbol{\Phi})\boldsymbol{x}_t + (\tilde{\boldsymbol{M}}_1 - \boldsymbol{M}_1)z_{t+1} + (\tilde{\boldsymbol{M}}_0 - \boldsymbol{M}_0)z_t\right]
\end{aligned}$$

then **the Risk-Neutral dynamics is a regime-switching VAR(1) process**:

$$\tilde{\boldsymbol{x}}_{t+1} = \tilde{\boldsymbol{M}}_1' z_{t+1} + \tilde{\boldsymbol{M}}_0 z_t + \tilde{\boldsymbol{\Phi}}\tilde{\boldsymbol{x}}_t + \Sigma^{1/2}(z_{t+1}, z_t)\varepsilon_{t+1} \quad (3)$$

with a homogeneous Markov chain on z_t

$$\mathbb{P}(z_{t+1} = e_j|x_t, z_t = e_i) = \tilde{\pi}_{ij} \quad (4)$$

Pricing forwards

- **Pricing of a unitary forward price of the *n*th commodity** (the simple case with no seasonality part $s_{n,t} = \ln S_{n,t}$ and $\tilde{x}_t = x_t$)

$$\begin{aligned} F_n(t,T) &= \mathbb{E}_t^{\mathbb{Q}}[S_{n,T}] \\ &= \mathbb{E}_t^{\mathbb{Q}}[\exp\{\tilde{s}_{n,T}\}] = \mathbb{E}_t^{\mathbb{Q}}\left[\exp\left\{e_n'\tilde{w}_T\right\}\right] \end{aligned}$$

with $\tilde{w}_t = (\tilde{s}_{1,t}, \ldots, \tilde{s}_{N,t}, \tilde{\delta}_{1,t}, \ldots, \tilde{\delta}_{N,t}, r_t, z_t)' = (\tilde{x}_t, z_t)'$

- **Recurrence formula on Laplace transforms**

Proposition 2 [Monfort & Féron, 2012]

If $\tilde{w}_t$ follows the Risk-neutral dynamics defined by (3) and (4), we have:

$$\mathbb{E}_t^{\mathbb{Q}}\left[\exp\left\{e_n'\tilde{w}_T\right\}\right] = \exp\{c_{n,h}'\tilde{w}_t\}$$

where $h = T - t$ and c_h are defined by recurrence:

$$\begin{aligned} c_{n,1} &= a(e_n) \\ c_{n,h} &= a(c_{n,h-1}) \end{aligned}$$

Internal consistency conditions

- For any storable commodity, the unitary forward price $F(t, t+1)$ can be obtained by two different ways:
  - from the Risk-neutral dynamics (Proposition 2)

$$F_n(t, t+1) = \exp\{a(e_n)'\tilde{\boldsymbol{w}}_t\}$$

  - from the historical dynamics

$$F_n(t, t+1) = S_{n,t} \exp\{r_t - \delta_t\}$$

- This leads to constraints in the recurrence function a and then in the Risk-Neutral parameters (i.e. **internal consistency conditions**).

Pricing forwards products and options

- **Possible extensions**
  - Seasonality parts (may be different in the historical and the risk-neutral probability)
  - increasing the auto-regression order to get VAR(m) (m can be different in the historical and risk-neutral dynamics)
  - **Forward product** price (in the case of non-storable commodity), with a delivery period θ

$$F_n^{theor}(t, T, \theta, \tilde{\boldsymbol{p}}) = \frac{1}{\theta} \sum_{T' \in [T \; ; \; T+\theta]} F_n(t, T, \tilde{\boldsymbol{p}})$$

  the unitary forward price $F_n(t, T)$ being a function of the risk-neutral dynamics parameters $\tilde{\boldsymbol{p}}$, the latent variables z_t and $\delta_{n,t}$, and the observed $s_{n,t}$.

- **Pricing European options** (Call on spot, Call on forward, Spread options) : using the Truncated Laplace transform [Monfort & Féron, 2012]

Estimation

- **Historical dynamics:** variable (s_t, z_t)

$$\boldsymbol{x}_{t+1} = \boldsymbol{M} z_{t+1} + \boldsymbol{\Phi}(x_t - \boldsymbol{M} z_t) + \Sigma^{1/2}(z_{t+1}, z_t)\varepsilon_{t+1}$$

$$P(z_t = e_j | z_{t-1} = e_i) = \pi_{ij}$$

- **Risk-neutral dynamics:**

$$\boldsymbol{x}_{t+1} = \tilde{\boldsymbol{M}}_1' z_{t+1} + \tilde{\boldsymbol{M}}_0 z_t + \tilde{\boldsymbol{\Phi}} x_t + \Sigma^{1/2}(z_{t+1}, z_t)\varepsilon_{t+1}$$

$$\mathbb{P}(z_{t+1} = e_j | x_t, z_t = e_i) = \tilde{\pi}_{ij}$$

- **Main issue for the estimation of observed spot prices and forward products**
  - Discrete latent variable z_t
  - Continuous latent variable $\delta_{n,t}$

Estimation: proposed strategy

Estimation procedure

1. Estimate the historical seasonalities $\nu_{n,t}$
2. Initialize the convenience yields $\delta_{n,t}$ by inverting the formula:

$$F_n(t,T) = S_{n,t} \exp\left\{(r_t - \delta_{n,t})(T-t)\right\}$$

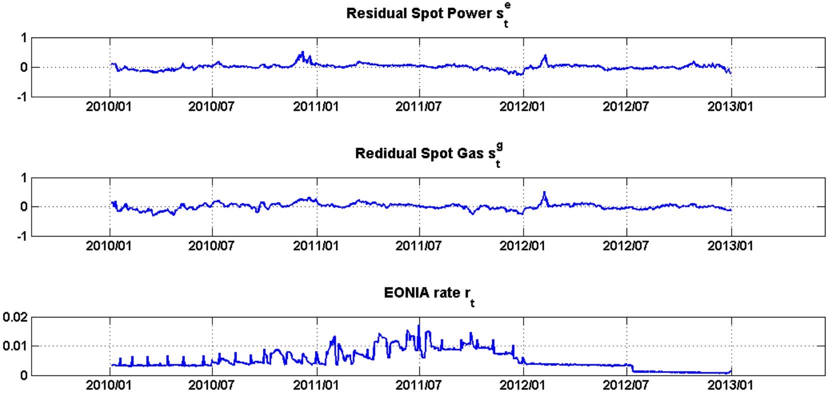
3. Estimate the historical parameters on observed spot prices by Maximum Likelihood criterion using a Kitagawa-Hamilton filter (which also gives an estimation of z_t)
4. Estimate the Risk-neutral dynamics on the N_f observed forward products by a non-linear least-square criterion

$$J(\tilde{\boldsymbol{p}}) = \sum_{t=1}^{T_f} \sum_{i=1}^{N_f} \left[\log F_{obs}(t, T_i, \theta_i) - \log F_{theor}(t, T_i, \theta_i, \tilde{\boldsymbol{p}})\right]^2$$

   using a (extended) Kalman filter (which also gives an estimation of $\delta n, t$)
5. return to step 3 until a given stoping criterion

The case with two commodities, K=2, m=1

- Historical data From 2010/01/01 to 2012/12/31
  UK Power (**non-storable**) spot prices, NBP Gas (**storable**) spot prices, EONIA (1day) interest rate

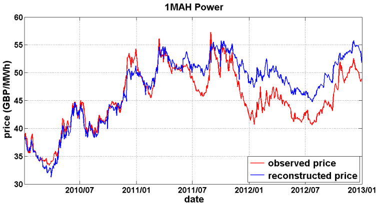


UK Power forwards: 1MAH $\rightarrow$ 3MAH, 1QAH $\rightarrow$ 4QAH, 1SAH $\rightarrow$ 6SAH

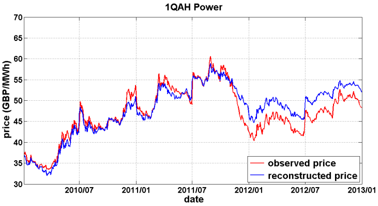
NBP Gas forwards: 1MAH $\rightarrow$ 3MAH, 1QAH $\rightarrow$ 4QAH, 1SAH $\rightarrow$ 6SAH

Results on forward reconstruction (not forecasting!)

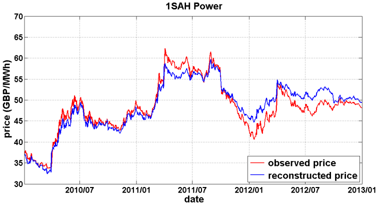
(a) Power 1-Month-ahead



(b) Power 1-Quarter-ahead



(c) Power 1-Season-ahead



(d) Power 6-Season-ahead

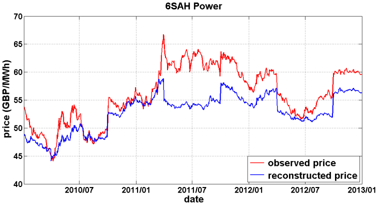
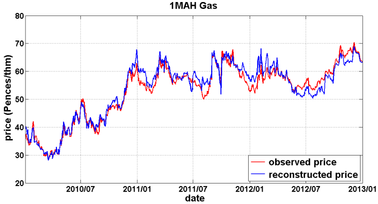


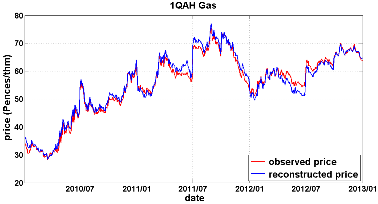
Figure: Results of forward reconstruction: realized forward price (red line) *versus* reconstructed forward prices (blue line) for the 3 years of calibration.

Results on forward reconstruction (not forecasting!)

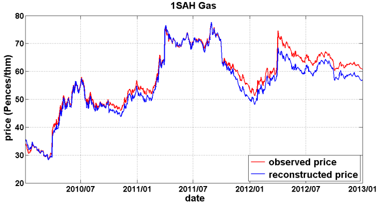
(a) Gas 1-Month-ahead



(b) Gas 1-Quarter-ahead



(c) Gas 1-Season-ahead



(d) Gas 6-Season-ahead

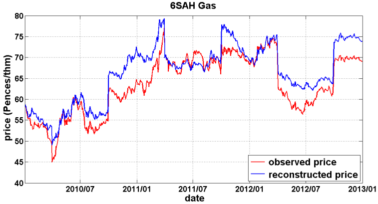


Figure: Results of forward reconstruction: realized forward price (red line) *versus* reconstructed forward prices (blue line) for the 3 years of calibration.

Conclusion and perspectives

Conclusion and perspectives

- **Conclusion**
  - Main contribution: propose an econometric approach for joint spot and forward prices, allowing quasi explicit formulas for European options
  - Promising results in the Electricity spot prices (capturing spikes), forward and options [Monfort & Féron, 2012]
  - Promising reconstruction results in the multivariate case
- **Perspectives**
  - The real case of multiple commodities (including Carbon for spark spread options)
  - improve the model and estimation procedure (to capture spikes)
    - for estimating a spike regime
    - for studying the estimation procedure (robustness...)

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