

# Thermal and nuclear energy portfolio selection using stochastic LCOE risk measures

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SMSA 2015/ Energy Finance Christmas 2015  
Wrocław, Feb. 19-20, 2015

# Overview

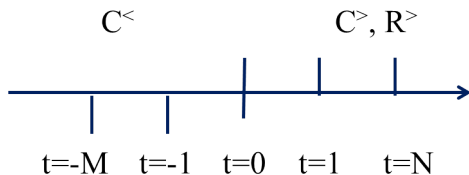
- 1 Deterministic vs Stochastic LCOE
- 2 Stochastic LCOE and Markowitz variance-cost tradeoff
- 3 Variance vs. CVaR as a Markowitz risk measure

# PV Equilibrium

In case of a cash flow with past  $t < 0$  and future  $t \geq 0$  sums  $S(t)$  and compound interest at given rate  $r_{cc}$  (**cost of capital**), **present value (PV)** equilibrium at  $t = 0$  is found when

$$\sum_{t < 0} \frac{S(t)}{(1+r)^t} = \sum_{t \geq 0} \frac{S(t)}{(1+r)^t},$$

$$t \in (-M, N).$$



# Breakeven Condition

In the case of building an **energy generation plant**, if the PV **at operations'** **starting time**  $t = 0$  of building **costs**  $C^<(t)$  ( $t < 0$ ) and operation **costs**  $C^>(t)$  ( $t > 0$ ) is lower than the PV of the sales **revenues**  $R^>(t)$  ( $t > 0$ ) as

$$C = \sum_{t < 0} \frac{C^<(t)}{(1 + r_{cc})^t} + \sum_{t \geq 0} \frac{C^>(t)}{(1 + r_{cc})^t} \leq \sum_{t \geq 0} \frac{R^>(t)}{(1 + r_{cc})^t} = R,$$

the project is **profitable**:  $C \leq R$ .

Equilibrium condition

$$C = R$$

is called **breakeven**.

# LCOE

Notice that profitability is expressed in terms of **price**  $\times$  **quantity levels**  $C$  and  $R$ .

In energy finance, the **Levelised Cost of Electricity (LCOE)** is a **constant price**  $\tilde{L}$  that helps to assess a **threshold for profitability** of building a new plant, when a **stream of quantities**  $Q^>(t)$  of electricity is sold.

# (Nominal) LCOE Implicit Definition

Assuming that the proceeds come from selling at the **constant price**  $\tilde{L}$  all electricity produced per period  $Q^>(t)$ ,

$\tilde{L}$  is implicitly **defined** as

$$C = \sum_{t < 0} \frac{C^<(t)}{(1 + r_{cc})^t} + \sum_{t \geq 0} \frac{C^>(t)}{(1 + r_{cc})^t} = \sum_{t \geq 0} \frac{\tilde{L} \times Q^>(t)}{(1 + r_{cc})^t} = R(\tilde{L}).$$

The **LCOE price**  $\tilde{L}$  is then a **breakeven price**.

# WACC

The effect of **financing** the project **by equity and debt** (minus **taxes**) can be included replacing the **cost of capital**  $r_{CC}$  with the **weighted average cost of capital** (WACC) rate

$$r_{wacc} \geq r_{CC}.$$

WACC, or WACC discount factors

$$F^W(t) = F_{t_0, t_n}^W = 1 / (1 + r_{wacc})^{t_n - t_0}$$

quantify the **perception of project riskiness** by equity and bond investors.

# Solving for price $\tilde{L}$

Setting

$$D = \sum_{t \geq 0} Q^>(t) F^W(t),$$

$$\tilde{L} = \frac{\sum_{t < 0} C_P^<(t) F^W(t)}{D} + \frac{\sum_{t \geq 0} C_P^>(t) F^W(t)}{D} = \tilde{I} + \tilde{O}$$

where  $\tilde{I}$  stands for normalized investments and  $\tilde{O}$  for normalized operation expenses, i.e. **nominal prices**.

If the electricity selling price  $E$  is such that

$$E \leq \tilde{L} = \tilde{I} + \tilde{O}$$

the project is **at loss**.



# Assumption of Constant $Q$

It will be assumed that the **electricity production**  $Q^>(t) = Q$  is **constant** (or known in advance).

Moreover, investment costs (i.e. costs at  $t \in (-M, -1)$ ) can be absorbed in operating costs, to give a **total nominal price**  $T$  such that

$$\tilde{L} = \tilde{I} + \tilde{O} = \tilde{T}.$$

# Real LCOE $L$

Taxes at rate  $T_c$  and **expected inflation**  $i$  (besides capital depreciation  $A^>(t)$  and expected real escalation rate  $\gamma$ , a sort of inflation for specific costs) can all be included in the calculation of a **real nominal price**  $T$  and a **real LCOE**  $L$ .

**Our real LCOE valuation formula for a single-fuel technology x plant is then**

$$L^x = \frac{\sum_{n=1}^M C_t F_{t_0, t_n}^W}{Q \sum_{n=1}^M (1+i)^{t_n-t_0} F_{t_0, t_n}^W} + \frac{I_0^W - T_c \sum_{n=1}^M dep_t F_{t_0, t_n}^W}{(1-T_c) Q \sum_{n=1}^M (1+i)^{t_n-t_0} F_{t_0, t_n}^W}.$$

# LCOE is Deterministic

Making the **time-varying fuel prices**  $X(t)$  and the **CO<sub>2</sub> emission prices**  $Z(t)$  explicit in costs  $C^>(t)$ ,

$$L^x = T(X(t), Z(t)), \quad t \in (0, N).$$

Practically, **expected values**  $\bar{X}$  and  $\bar{Z}$  are used:

$$L = T(\bar{X}, \bar{Z}).$$

This implies that their time series are assumed to be **constant** and to have **zero volatility**. Then,

**$L$  has zero volatility.**

# LCOE goes Stochastic

Assume  $\omega \in \Omega$  to be a **path extraction** from the **event set**  $\Omega$ .

If fuel and  $CO_2$  nominal prices  $X(t)$  and  $Z(t)$  are replaced by **stochastic processes**  $X_\omega(t)$  and  $Z_\omega(t)$ ,

$L$  becomes a **time independent stochastic variable**

$$L_\omega = T(\hat{X}_\omega, \hat{Z}_\omega).$$

since the component stochastic processes are **weighted** and **summed** over their paths.

A mutual **dependency structure** can be chosen for  $X_\omega(t)$  and  $Z_\omega(t)$ , for example imposing some correlation.

Economically, this would mean a correlation between fuel prices and  $CO_2$  prices.

$L_\omega$  has now a **distribution**, a **mean**  $\mu_L$  and a **variance**  $\sigma_L^2$ .

For any given  $E$ , there can be some  $\omega$  such that

$$E \leq L_\omega.$$

This models investment **risk**.

**Factor modelling** is included in this approach.

Since fuel and related  $CO_2$  cost enter  $L_\omega$  in a **linear** way, for a single-fuel plant

$$L_\omega = I + O^x(\hat{X}_\omega) + O^{x-CO_2}(\hat{Z}_\omega) = I + O_\omega^x + O_\omega^{x-CO_2}$$

where  $O$  are normalized component costs.

$O_\omega^{x-CO_2}$  is proportional to  $\hat{Z}_\omega$  through the **fuel  $CO_2$  intensity**  $f^{fuel}$ :

$$O_\omega^{x-CO_2} = f^x A \hat{Z}_\omega.$$

Then, for a single-fuel plant,

$$E[L_\omega] = \mu_L^x = I + \mu^x + \mu^{x-CO_2}.$$

For two different fuels, say coal  $X$  and gas  $Y$ ,

$$L_{\omega}^{\text{coal}} = I + O_{\omega}^{\text{coal}}(\hat{X}_{\omega}) + O_{\omega}^{\text{coal-CO}_2}(\hat{Z}_{\omega}) = I + O_{\omega}^{\text{coal}} + O_{\omega}^{\text{coal-CO}_2}$$

$$L_{\omega}^{\text{gas}} = I + O_{\omega}^{\text{gas}}(\hat{Y}_{\omega}) + O_{\omega}^{\text{gas-CO}_2}(\hat{Z}_{\omega}) = I + O_{\omega}^{\text{gas}} + O_{\omega}^{\text{gas-CO}_2}$$

so that coal and gas LCOE share a **factor** in  $\hat{Z}_{\omega}$ , common to  $O_{\omega}^{\text{coal-CO}_2}$  and  $O_{\omega}^{\text{gas-CO}_2}$ .

If  $X_\omega$  and  $Y_\omega$  are **independent** (then they must be independent from  $Z_\omega$  too),

$$E[(L_\omega^{\text{coal}} - I) (L_\omega^{\text{gas}} - I)] = \mu^{\text{coal}} \mu^{\text{gas}} + \mu^{\text{coal}} \mu^{\text{gas-CO}_2} + \mu^{\text{coal-CO}_2} \mu^{\text{gas}} + \alpha^{\text{CO}_2}$$

where

$$\alpha^{\text{CO}_2} = E[O_\omega^{\text{coal-CO}_2} O_\omega^{\text{gas-CO}_2}] \neq 0,$$

so that, in terms of covariance,

$$\text{COV}((L_\omega^{\text{coal}} - I), (L_\omega^{\text{gas}} - I)) = \alpha^{\text{CO}_2} = A^2 I^{\text{coal}} I^{\text{gas}} \sigma^2(\hat{Z}_\omega).$$



If one of the processes is **constant**, for example assuming that the fuel price  $N_\omega = N_0$  for **nuclear plants** has very small volatility, and has **zero emissions** ( $Z_\omega = 0$ ),

$$\sigma_L^{\text{nuclear}} = 0$$

$$\mu_L^{\text{nuclear}} = N$$

$$\text{COV}((L_\omega^{\text{nuclear}} - I), (L_\omega^{\text{coal}} - I)) = 0,$$

$$\text{COV}((L_\omega^{\text{nuclear}} - I), (L_\omega^{\text{gas}} - I)) = 0.$$

# Portfolio Stochastic LCOE

Recap. For **multi-technology generation portfolios**,

- since **fossil fuels** share a  $\text{CO}_2$  factor, a **portfolio stochastic LCOE** is the sum of **correlated** LCOEs,
- even though the fuels' process prices are taken independent.
- A **nuclear fuel** LCOE can be taken **deterministic**.

# Simulation of a coal or gas portfolio

The coal  $X$ , gas  $Y$  and  $\text{CO}_2$   $Z$  **nominal price processes** are assumed to be **geometric brownian motions discretized** on a **yearly grid**

$$dX = (\mu_X + \pi)X dt + \sigma_X X dW_X$$

$$dY = (\mu_Y + \pi)Y dt + \sigma_Y Y dW_Y$$

$$dZ = \pi Z dt + \sigma_Z Z dW_Z$$

where  $\mu_X$  and  $\mu_Y$  are functions of expected real escalation rates  $\gamma$ ,  $\pi = \ln(1 + \iota)$  is a function of the expected inflation rate  $\iota$ ,  $\sigma_X$ ,  $\sigma_Y$  and  $\sigma_Z$  volatilities of **independent Wiener processes**  $W_X$ ,  $W_Y$  and  $W_Z$ .

Notice that

- 1) fuel prices are independent from each other,
- 2) fuel prices are not correlated to CO<sub>2</sub> prices,
- 3) the **real prices** too follow a very simple, lognormal process

$$dX = \mu_X X dt + \sigma_X X dW_X$$

$$dY = \mu_Y Y dt + \sigma_Y Y dW_Y$$

$$dZ = \sigma_Z Z dW_Z$$

# The **parameters** used to wrap the price processes and compute the **LCOE** for a plant in the US are

	Units	Nuclear	Coal	Gas
Nominal capacity	MW	2236	1300	540
Capacity factor		90%	85%	87%
Heat rate	Btu/kWh	10460	8800	7050
Overnight cost	\$/kW	5335	2844	978
Fixed O&M costs	\$/kW/year	88.75	29.67	14.39
Variable O&M costs	mills/kWh	2.04	4.25	3.43
Fuel costs	\$/MMBtu	0.71	2.26	5.14
Fuel's CO <sub>2</sub> intensity	Kg-C/MMBtu	0	25.8	14.5
Waste fee	\$/kWh	0.001	–	–
Decommissioning cost	\$ million	750	–	–
O&M real escalation rate		1.0%	1.0%	1.0%
Fuel real escalation		0.5%	0.9%	1.4%
Construction period	years	6	4	3
Operations		2018	2018	2018
Plant life	years	40	40	40
Depreciation schedule		MACRS,15	MACRS,20	MACRS,15

Start of operation ( $t = 0$ ) is 2012. Overnight costs are assumed to be uniformly distributed on the construction period. Depreciation is developed according to the MACRS.

(all **real** year 2010 costs) and

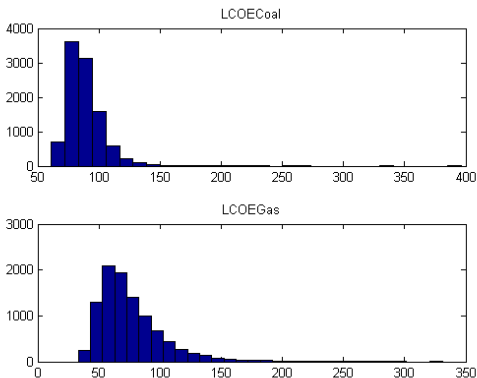
$$\begin{aligned} \iota_{\text{inflation}} &= 1.8\% & \text{WACC} &= 7\% \\ & & T_c &= 40\% \end{aligned}$$

After estimating  $\sigma$  and  $\gamma$  from market time series, the **parameters** for the **price processes** become

$$\begin{aligned} \gamma_c &= 0.9\% & \gamma_g &= 1.4\% \\ \sigma_c &= 0.09 & \sigma_g &= 0.16 \\ \sigma_{\text{CO}_2} &= \text{scenario for 10 yrs,} & & \text{then 0.15} \\ \mu_c &= \ln(1.009) & \mu_g &= \ln(1.014) \\ \pi &= \ln(1.018) & & \end{aligned}$$

# Scenario Analysis

Real LCOE distribution, for a **single technology** coal and a gas plant.



For two **single-fuel** plants  $\mu_L^{\text{fuel}}$  is estimated in **dollars** under **three CO<sub>2</sub> volatility scenarios**.

	$\sigma_Z = 0.2$	$\sigma_Z = 0.3$	$\sigma_Z = 0.4$
$\mu_L^{\text{coal}}$	88.2	88.2	88.2
$\mu_L^{\text{gas}}$	76.2	76.2	76.2
$\sigma_L^{\text{coal}}$	14.5	21.7	32.7
$\sigma_L^{\text{gas}}$	28.2	29.2	31.2
$\rho$	0.18	0.30	0.45

Notice 1) the nonzero correlation coefficient  $\rho$ , due to the CO<sub>2</sub> common factor, 2) the inversion of LCOE riskiness between coal and gas as  $\sigma_Z$  increases, 3) correlation  $\rho$  increases with CO<sub>2</sub> volatility.



# LCOE Portfolio Theory

Now **LCOE itself** can be used in the Markowitz risk-return optimization.

In this case,  $-\mu_L$  plays the role of the **expected return** (you want  $L_\omega$  low), and its variance plays the role of the **risk**.

The **minimum attainable variance** is  $\sigma_{L,m}^2 = (\sigma_L^{\text{multifuel,minimum}})^2$ , and the **optimal LCOE** becomes  $\mu_{L,m} = \mu_L^{\text{multifuel,minimum}}$ .

Only modification, either the **efficient frontier becomes the lower one**, or  $-\mu_L$  is used.

For one **bi-fuel** plant, **minimum variance LCOE portfolio weights**  $w_m^{\text{coal}}$  and  $w_m^{\text{gas}}$ , and optimal  $\mu_{L,m}$ , are estimated under **three CO<sub>2</sub> volatility scenarios**.

	$\sigma_Z = 0.2$	$\sigma_Z = 0.3$	$\sigma_Z = 0.4$
$w_m^{\text{coal}}$	84.0%	70.2%	45.5%
$w_m^{\text{gas}}$	16.0%	29.8%	54.5%
$\mu_{L,m}$	86.3	84.6	81.7
$\sigma_{L,m}$	13.8	19.7	27.2

Notice the inversion of relative weight between coal and gas as  $\sigma_Z$  increases. Increasing CO<sub>2</sub> volatility makes coal generation riskier.

For one **tri-fuel** plant which includes **nuclear**, the portfolio can benefit of a **risk-free** asset, with the **highest cost** (i.e. lowest rate), with an extra weight  $w^{\text{nuclear}}$ .

At the same  $\mu_{L,m}$  **found before** for the bi-fuel plant,

	$\sigma_Z = 0.2$	$\sigma_Z = 0.3$	$\sigma_Z = 0.4$
$\mu_{L,m}$	86.3	84.6	81.7

the riskiness of the portfolio can now be **reduced**.

$w_e^{\text{coal}}$ ,  $w_e^{\text{gas}}$ ,  $w_e^{\text{nucl}}$  will be the new **efficient frontier weights**.

$\mu_L^{\text{nuclear}} = 95.4$	$\sigma_X = 0.2$	$\sigma_X = 0.3$	$\sigma_X = 0.4$
$\mu_{L,\text{trifuel}}$	86.3	84.6	81.7
$\sigma_{L,m} \text{ (bifuel)}$	13.8	19.7	27.2
$\sigma_{L,\text{trifuel}}$	11.7	16.0	22.3
$W_e^{\text{coal}}$	39.8%	15.9%	0.0%
$W_e^{\text{gas}}$	32.6%	50.2%	71.6%
$W_e^{\text{nucl}}$	27.6%	33.9%	28.4%

Notice that the included coal generation goes to zero as  $\sigma_Z$  increases, in contrast to gas generation which increases. Nuclear remains constant.

For a  $\mu_L^{\text{nuclear}} = N = 95.4$  it turns out that,

**as  $\sigma_Z$  increases,**

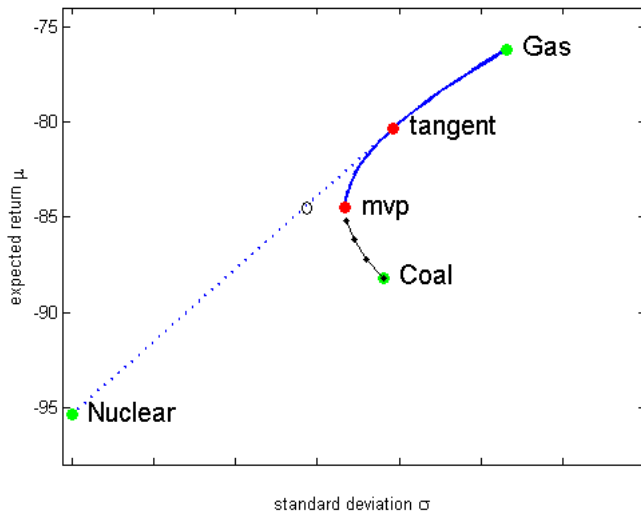
- same as seen for the bi-fuel plant, **investment riskiness  $\sigma_{L,\text{trifuel}}$  increases,**
- **but it is always lower** than the the bi-fuel case  $\sigma_{L,m}$  (bifuel),
- and the **gain in risk reduction is larger and larger.**

This means that **investment riskiness can be reduced** by **inclusion of nuclear** generation.

The **tangent portfolio** is **Sharpe-optimal**. For **larger values** of  $N$ , this effect is weaker.

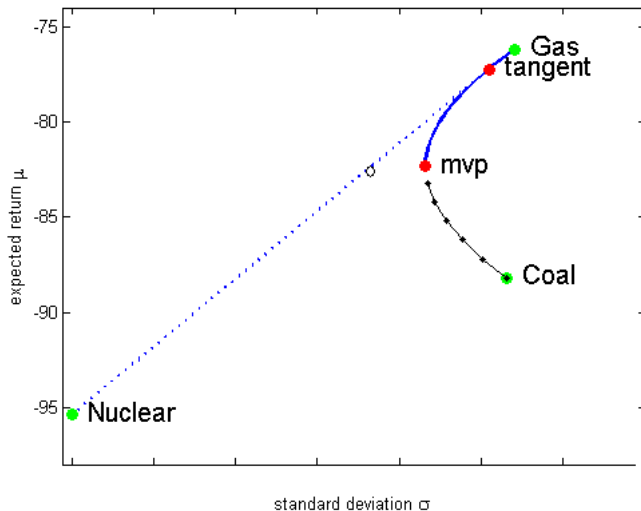
# Scenario Analysis: $\sigma_{CO_2} = 0.2$

$\mu$ - $\sigma$  frontiers for  $\sigma_{CO_2} = 0.20$



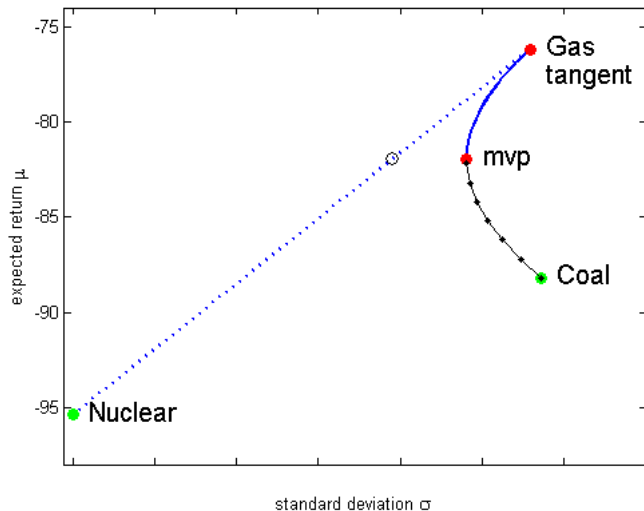
# Scenario Analysis: $\sigma_{CO_2} = 0.3$

$\mu$ - $\sigma$  frontiers for  $\sigma_{CO_2} = 0.30$



# Scenario Analysis: $\sigma_{CO_2} = 0.4$

$\mu$ - $\sigma$  frontiers for  $\sigma_{CO_2} = 0.40$





## Recap. Notice that

- 1 at fixed CO<sub>2</sub> riskiness, without nuclear, if attitude is risk-prone, the only-gas solution should be chosen. That would minimize expected LCOE.
- 2 At fixed CO<sub>2</sub> riskiness, without nuclear, the theory gives indications about the best mix of thermal technologies to be used - the minimum variance portfolio,
- 3 at fixed CO<sub>2</sub> riskiness, addition of a nuclear component can further decrease risk.
- 4 As CO<sub>2</sub> riskiness increases, gas appetibility increases but gain due to nuclear increases as well.
- 5 As nuclear price increases, this effect becomes weaker.
- 6 The tangent portfolio gives the optimal Sharpe risk/return ratio.

# Environmental Trade-off

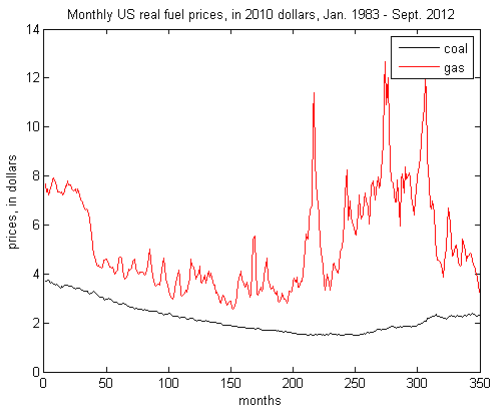
Notice that **WACC** was kept constant during the analysis, and that the LCOE is strongly nonlinear in WACC levels.

WACC can include the risk perception of investors in the nuclear business.

**Including WACC** in the **scenarios** can help to understand how **two environmental risks**, the nuclear business risk and CO<sub>2</sub> prices volatility, **compete** in the decision of setting how much weight to give to nuclear assets in the energy portfolio, when **scenario risk minimization** is sought. This is left for further work.

# Extended Model

The US doesn't have a CO<sub>2</sub> market, then a lognormal model can be safely assumed. But models of coal and gas prices can be improved.



**Mean reverting log-processes** are chosen for the real coal and gas prices, and **jumps** are included in the gas process.

$$d\hat{X} = (\mu_X - \theta_X \hat{X}) dt + \sigma_X dW_X$$

$$X = \exp \hat{X}$$

$$d\hat{Y} = (\mu_Y - \theta_Y \hat{Y}) dt + \sigma_Y dW_Y + J(\sigma_J) dN(\lambda)$$

$$Y = \exp \hat{Y}$$

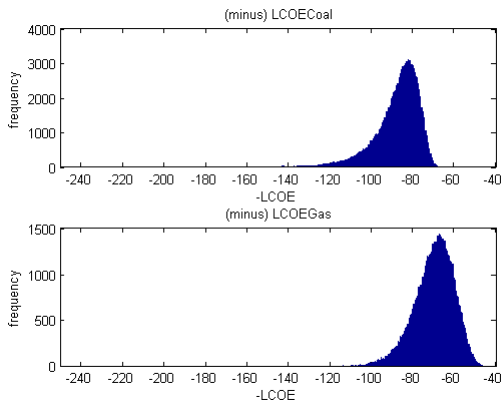
$$dZ = \pi Z dt + \sigma_Z Z dW_Z$$

These series are **discretized** on a monthly time grid, and assumed as models for the true price dynamics.

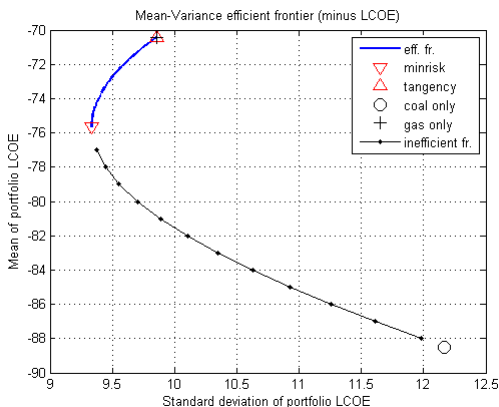
After estimating  $\sigma$  and  $\gamma$  from **monthly** market time series, the **monthly parameters** for the **real price processes** (which will be then subject to escalation and inflation on a monthly base) become

$\mu_c = 0.0000$	$\mu_g = 0.0292$
$\theta_c = 0.0000\%$	$\theta_g = 0.0210$
$\sigma_c = 0.0121$	$\sigma_g = 0.0602$
$\lambda = 0.2684$	$\sigma_j = 0.1258$
$\sigma_{CO_2} = \text{scenario}$	(but only $\sigma = 0.2$ here)

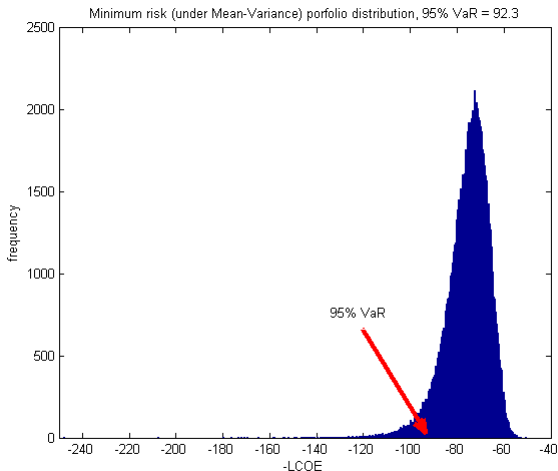
Case  $\sigma_{CO_2} = 0.2$ . The LCOE distributions (horizontally **flipped**) develop a **thick, long tail** on the **undesirable side** (high LCOE values).



In the Mean-Stdv plane, the **efficient frontier** includes **low value** LCOE portfolios. On the tip of the Markowitz bullet, the **minimum risk portfolio**, a  $w_c = 0.2885$ ,  $w_g = 0.7115$  combination of coal and gas plants.

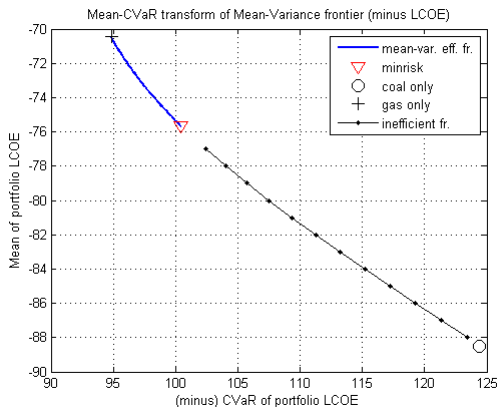


The minimum risk portfolio 95% VaR is \$ 92.3, but 95% **CVaR** is \$ 100.5, ten percent higher. There is a **lot of downside scenario risk** in the tail.

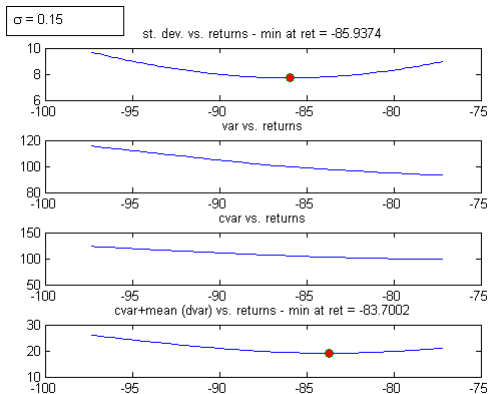




The Mean-CVaR plane reveals a reversal of situation. If tail risk is undesirable, the best choice is an only-gas production, with a 95% CVaR of 95.0 dollars.



# A CVaR Deviation (DVaR) analysis offers a different perspective, for an investor averse to tail risk.



# Conclusions

A new approach to LCOE feasibility assessment was presented. Advantages:

- In its **lognormal version** which includes the nuclear option, it allows for an analysis to explore trade-offs between economic advantages and environmental issues, from a financial point of view.
- In its **extended version**, it allows for a risk assessment about using classic deterministic LCOE analysis to decide if and what plant to build.

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