Institute for Operations Research
and Computational Finance

University of St.Gallen

SMSA 2015 Statistics in Energy

Optimization of hydro storage systems and indifference pricing of power contracts

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Outlook

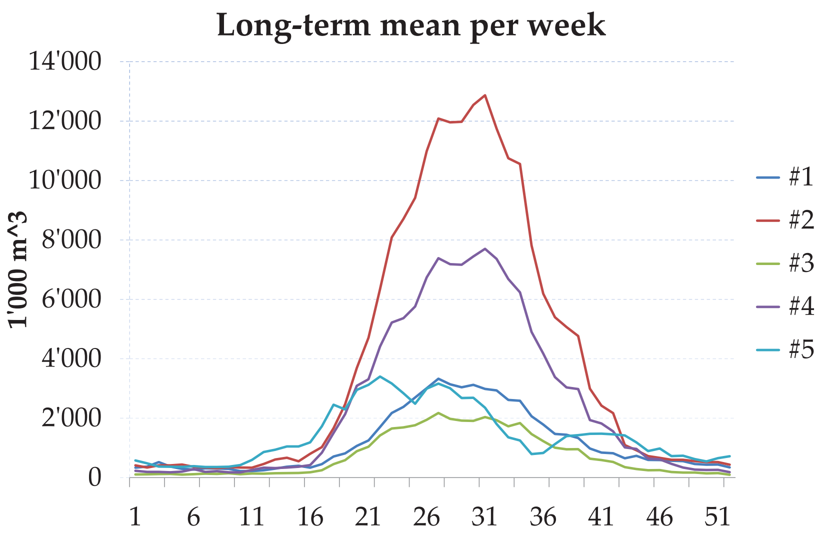
- Producer owns **pumped-storage hydropower plants** (PSHPs). These consist of large (seasonal) connected reservoirs
- We must decide on a production schedule
- We aim at a **mid-term planning model for hydropower production** based on **multistage stochastic programming**
  - Stochastic inflows; stochastic electricity prices
  - Produced electricity is sold at the spot market
- Valuation of non-standard power contracts with indifference pricing (work in progress)
  - Stochastic load profiles
  - Indifference price: entering the contracts with individual consumers or selling only at the spot market

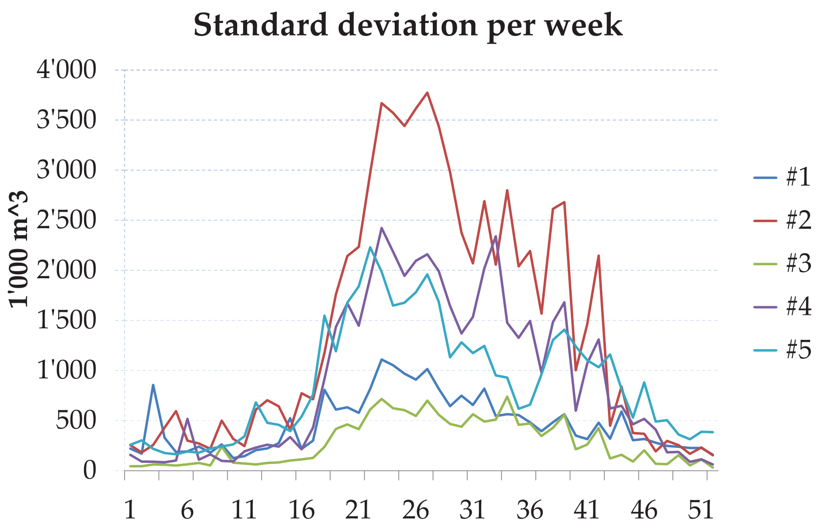
Inflow data

- Daily inflows are given for **a system with five connected reservoirs** (sample period: 26 years)
- Data are aggregated to weekly inflows
- The inflows show a characteristic seasonal pattern:
  - Winters are dry
  - Inflows are higher in the middle of the year
- Thus, **a seasonal adjustment will be performed**

Seasonality

- Let $f_i(\tau)$ be the inflow in reservoir $i = 1, \ldots, 5$ at time $\tau = 1, \ldots, T$
- Let $\bar{f}_i(w')$ long-term mean of the inflows, $w' = 1, ..., 52$ and $\hat{\sigma}_i(w')$ the standard deviations of the inflows after seasonal adjustment for all reservoirs over one year
- Allocate each time point to one week w': $\mathcal{T}(w') := \{\tau = 1, ..., T \mid w(\tau) = w'\}$
  - Long-term seasonal adjustment: $f_i^{adj}(\tau) = f_i(\tau) - \bar{f}_i(w(\tau))$
  - Correct for time-varying volatility: $f_i^{std}(\tau) = \frac{f_i^{adj}(\tau)}{\hat{\sigma}_i(w(\tau))}$





Principal component analysis

- The principal component analysis is performed on the stochastic component of the inflows
- According to the eigenvalue criterion, only the first factor should be considered
- Then 75% of the variation can be explained; with 2(3) factors we explain about 86(93) percent of the variation

Table 1: Principal component analysis

Component	Total	% of Variance	Cumulative %
1	3.744	74.9	74.9
2	0.573	11.5	86.3
3	0.371	7.4	93.8
4	0.279	5.6	99.3
5	0.033	0.7	100.0

Spot model for electricity prices

The formulation of the regime-switching model reads:

$$X_t = \begin{cases} \ell_t^L - \xi_t^-, & \text{if the system is in the lower spike regime,} \\ s_t \cdot \exp(r_t), & \text{if the system is in the base regime or} \\ \ell_t^U + \xi_t^+, & \text{if the system is in the upper spike regime.} \end{cases} \quad (1)$$

$$dr_t = -\kappa_\alpha r_t dt + \sigma_\alpha dZ_t \quad (2)$$

$$l_t^L = s_t / \exp(a_\gamma^L), \qquad l_t^U = s_t \cdot \exp(a_\gamma^U). \quad (3)$$

$$\xi_t^- \sim \text{Wei}(\lambda_\beta^-, k_\beta^-), \qquad \xi_t^+ \sim \text{Wei}(\lambda_\beta^+, k_\beta^+)$$

X_t	spot price in hour t
s_t	seasonality shape (deterministic component)
r_t	stochastic component of the spot price in the base regime
ξ_t^+	upward spike
ξ_t^-	downward spike
ℓ_t^U	upper limit of the base regime in hour t
ℓ_t^L	lower limit of the base regime in hour t

Transition probabilities and results

- The probabilities of remaining in the current regime or switching to another one from one hour to the next are modeled by a (time-dependent) transition matrix:

$$\Pi_\delta = \begin{pmatrix} 1-\pi_\delta^{LB} & \pi_\delta^{LB} & 0 \\ \pi_\delta^{BL} & 1-\pi_\delta^{BL}-\pi_\delta^{BU} & \pi_\delta^{BU} \\ 0 & \pi_\delta^{UB} & 1-\pi_\delta^{UB} \end{pmatrix}. \tag{4}$$

  where $\delta := \delta(t)$ is a function that maps t to a set of indices (seasonality)

- Estimation procedure: **two-step maximum likelihood**

Estimation results

Estimation sample: Hourly EEX spot prices from 1 Jan 2009 to 31 Dec 2013

Param.	a_γ^L	a_γ^U	κ_α	σ_α	λ_β^-	k_β^-	λ_β^+	k_β^+	$E(\xi_t^-)$	$E(\xi_t^+)$
Overall	0.697	0.875	2.848	0.566	0.127	0.737	0.09	0.798	9.493	12.681
			(0.05)	(0.001)	(0.005)	(0.01)	(0.018)	(0.101)		
Summer	0.649	0.894	3.045	0.555	0.184	0.936	0.105	1.326	5.591	8.759
			(0.075)	(0.002)	(0.008)	(0.015)	(0.042)	(0.899)		
Winter	0.89	1.054	2.538	0.606	0.101	0.646	0.065	1.134	13.657	14.771
			(0.057)	(0.002)	(0.006)	(0.015)	(0.027)	(0.542)		

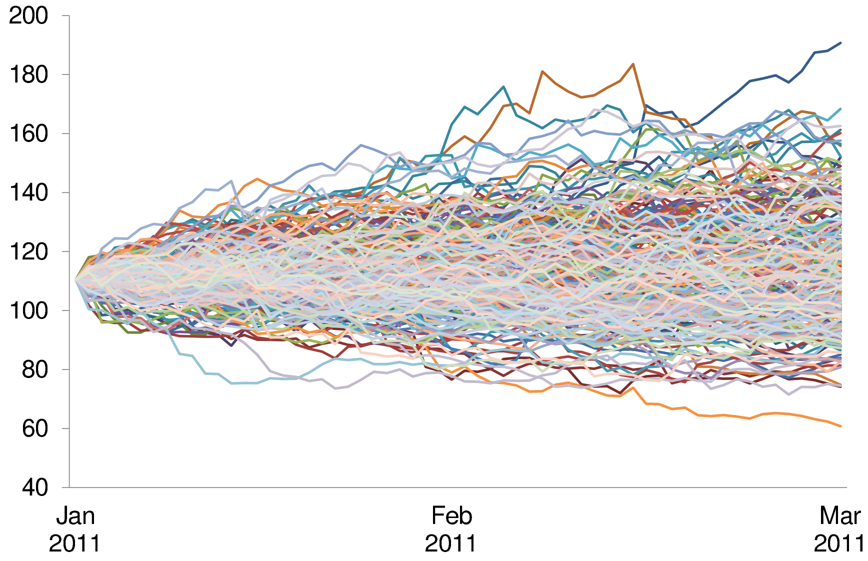
Table 2: Estimation results#1 (standard errors in parenthesis). $E(\xi_t^{+/-})$ shows the expected value of upwards/downwards spike magnitudes

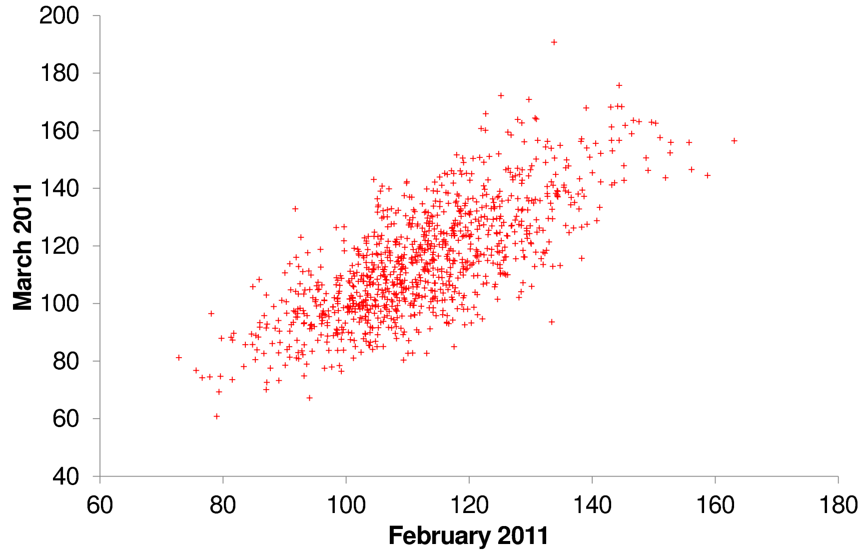
Param.	$\pi_\delta^{BL/LB}$	$\pi_\delta^{BU/UB}$	$\#base$	$\#lower$	$\#upper$	loglike
Overall	1.053%	0.078%	41937	1837	45	27156
	24.061%	73.333%				
Summer Base	1.631%	0.015%	21130	826	4	27338
	28.800%	66.666%				
Summer Peak	0.193%	0.013%				
	22.368%	100%				
Winter Base	1.309%	0.007%	21064	788	7	
	24.386%	100%				
Winter Peak	0.078%	0.064%				
	12.631%	80%				

Table 3: Estimation results#2 (standard errors in parenthesis)

Generating scenarios

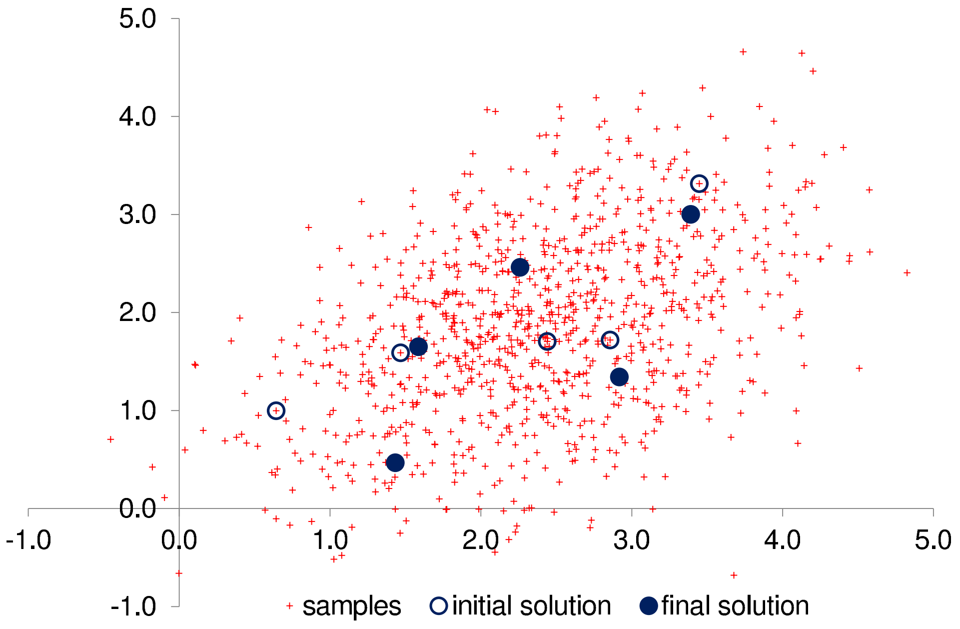
- We generate 10'000 scenarios in weekly steps for:
  - The stochastic component of the spot price process for the base regime
  - Two principal components of the inflow model
- Find a tree that is:
  - Small enough as a numerically tractable approximation
  - Large enough to capture important features of the problem
- The scenario tree should be as close as possible to the observed stochastic process





Excursus: Minimizing the distance for distributions

- Consider a random variable G that is either continuous or discrete (with a very large number of mass points)
- We want to approximate G by a simpler random variable $\tilde{G}$
- The distance between G and $\tilde{G}$ can be measured by the Wasserstein distance $d(G, \tilde{G})$
- It is known that the Wasserstein distance is related to a transportation problem
- The problem of minimizing the distance $d(G, \tilde{G})$ is solved by assigning data points to a few "clusters" (which represent the approximate distribution $\tilde{G}$)

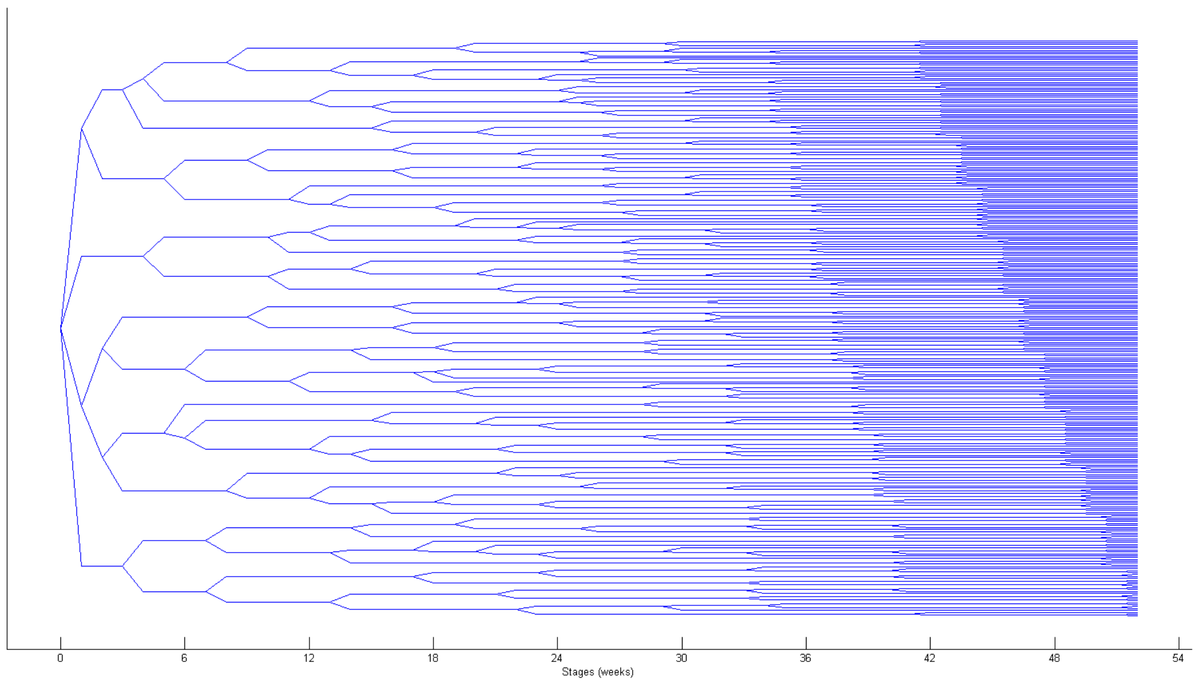


$$\mu = \begin{pmatrix} 2.37 \\ 1.83 \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} 0.71 & 0.33 \\ 0.33 & 0.91 \end{pmatrix}$$

Extension to nested distances

- Pflug/Pichler (2012) introduced and analyzed a generalization of the well known Wasserstein distance
- Kovacevic/Pichler (2012) propose an algorithm for improving the distance between the trees
- This *nested distance* takes the information from the filtration into account (rather than comparing only scenario paths)
- Based on this concept, the tree resulting from the first step is further improved by adjusting the probabilities and values



Time aggregation: price level approach

- On the one hand, the valuation of non-standard power contracts requires an hourly time resolution
- On the other hand, the computational effort would explode since scenario trees grow exponentially with the number of stages
- Thus, an **aggregation technique** is required!
- We apply the price level approach (a.k.a. occupation times)

Optimization model prerequisites

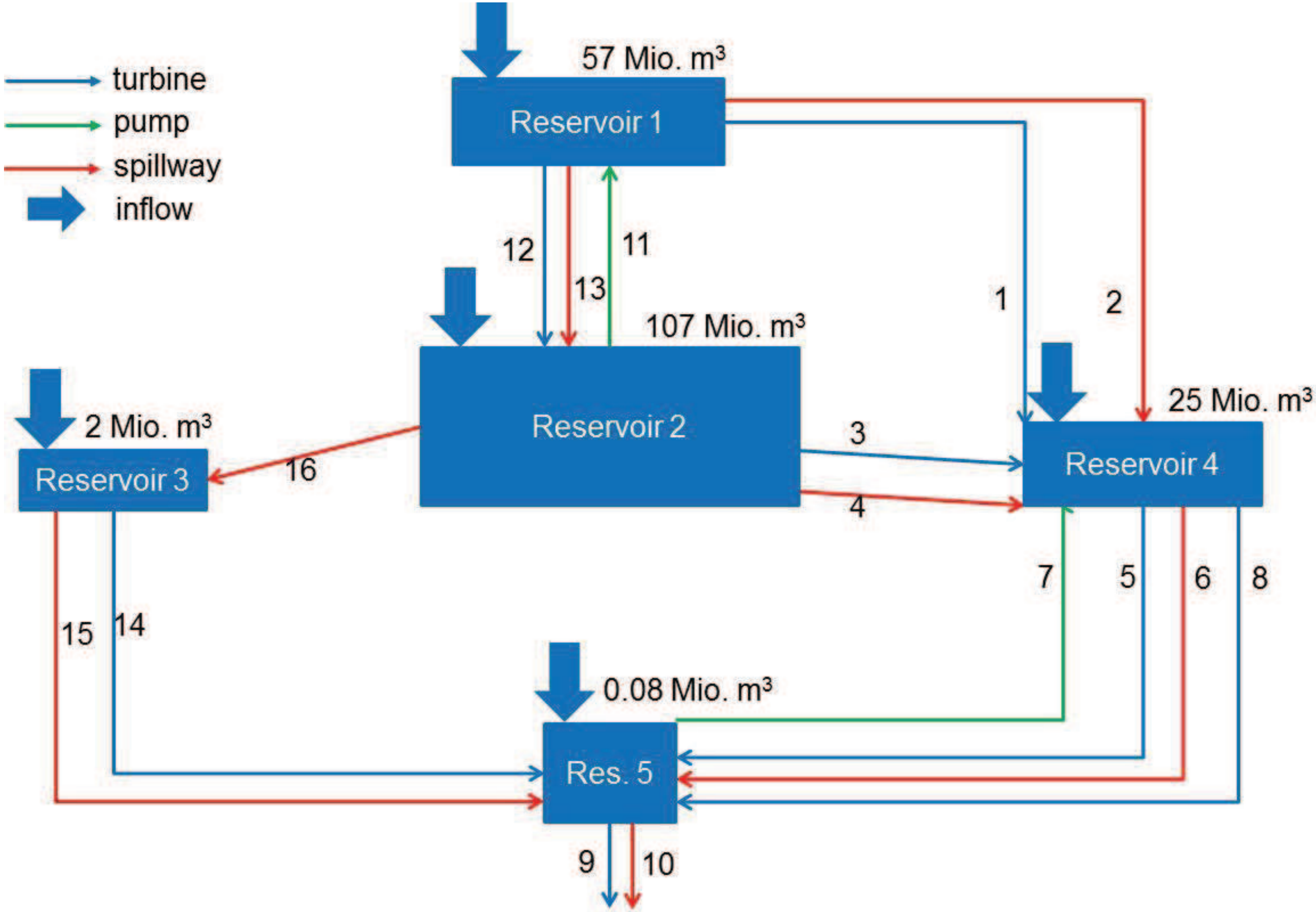
- Notation:

$$\begin{array}{ll} \mathcal{I} = \{1, \ldots, I\} & \text{set of reservoirs} \\ \mathcal{J} = \{1, \ldots, J\} & \text{set of connections between reservoirs} \\ \mathcal{T} \subset \mathcal{J} & \text{set of turbines} \\ \mathcal{P} \subset J & \text{set of pumps} \\ \mathcal{W} \subset J & \text{set of spillways} \\ \mathcal{L} = \{1, ...L\} & \text{price level index set} \end{array}$$

- The system topology is described by an incidence matrix A such that

$$A_{ji} = \begin{cases} 1 & \text{if water flows into reservoir } i \text{ over connection } j, \\ -1 & \text{if water flows out of reservoir } i \text{ over connection } j, \\ 0 & \text{if connection } j \text{ is not related to reservoir } i. \end{cases}$$

Reservoirs layout



System parameters

- **System parameters (deterministic):**

$\bar{s}_i$	maximum storage volume (minimum is zero) in m^3
g_i	minimum storage level (end of planning horizon in m^3
$\underline{q}_j, \bar{q}_j$	minimum/maximum water flow in m^3/h
η_j	production efficiency in MWh/m^3
$\underline{p}_j, \bar{p}_j$	lower/upper bounds on produced power in MWh

- **Remarks:**
  - Turbines have positive efficiency (conversion from water to electricity)
  - Pumps have negative efficiency, negative minimum power and maximum power of zero
  - Spillways have zero efficiency and nonnegative flow capacity

- **System parameters (stochastic):**

S_t	electricity spot price at time t in EUR/MWh
F_{it}	inflow in reservoir i (precipitation, snowmelt etc.) in m^3

Tree notation

- The (joint) stochastic process of the uncertain factors is represented by a finite tree with node set $\mathcal{N} = \{0, 1, ..., N\}$
- $\mathcal{N}_t$ is the set of nodes at level $t = 0, ..., T$
- The tree structure represents the filtration of the process and can be defined by stating the unique predecessor n^- for each node n
- In the tree-based model with price levels notation changes:

$S_{n\ell}$	spot price at level ℓ in node n
F_{in}	inflow in reservoir i in node n in m^3

- Further definitions:

Δ_n	length of planning period beginning in node n
π_n	probability of node n
$\pi_{n\ell}$	probability of price level ℓ in node n (index set: $\mathcal{L}$)

Optimization model

- For each price level ℓ at each node n the producer decides how much water $q_{jn\ell}$ flows over connection j (in m^3/h)
- This corresponds to the produced power $p_{jn\ell}$ (in MW) and energy $e_{jn\ell}$ (in MWh) over the period:

$$p_{jn\ell} = \eta_j q_{jn\ell}, \tag{5}$$

$$e_{jn\ell} = p_{jn\ell} \pi_{n\ell} \Delta_n \tag{6}$$

- Technical constraints for produced (consumed) power:

$$\underline{p}_j \leq p_{jn\ell} \leq \bar{p_j} \tag{7}$$

- The stored water s_{in} in each reservoir at the beginning of the period is restricted by the reservoir capacity:

$$0 \leq s_{in} \leq \bar{s}_i \tag{8}$$

- It starts from an initial value s_{i0} and develops over time as:

$$s_{in} = s_{in^-} + \sum_{j \in \mathcal{J}} A_{ji} \sum_{\ell \in \mathcal{L}} \pi_{n\ell} q_{jn^-\ell} \Delta_n + F_{in} \tag{9}$$

- At time T, the stored water volume has to meet the minimum level:

$$s_{in} \geq \bar{g}_i \quad (n \in \mathcal{N}_T) \tag{10}$$

Optimization model

- We start with an initial cash amount c_0; for later time points $n > 0$ cash evolves according to the budget equation:

$$c_n = R_{n^-} c_{n^-} + \sum_{j \in \mathcal{T} \cup \mathcal{P}} \sum_{\ell \in \mathcal{L}} e_{jn-\ell} S_{n-\ell} \tag{11}$$

  where R_n is the interest rate in node n over the period Δ_n (equal for lending and borrowing)

- **The objective is defined as a mix of expectation and average value at risk** in view of the available cash at the end of the planning horizon in T:

$$\max \lambda E(\text{“}cash\ in\ T\text{”}) + (1 - \lambda) AVaR(\text{“}cash\ in\ T\text{”}) \tag{12}$$

Optimization model overview

- The complete model in the tree-based notation is:

$$\max \lambda \sum_{n \in \mathcal{N}_T} \pi_n c_n + (1 - \lambda) z \tag{13}$$

$$
\begin{array}{ll}
s.t. & z \leq y_0 - \dfrac{1}{\alpha} \displaystyle\sum_{n \in \mathcal{N}_T} \pi_n y_n \\
n \in \mathcal{N}_T : & c_n - y_0 + y_n \geq 0 \\
n \in \mathcal{N}_T : & y_n \geq 0 \\
j \in \mathcal{J}; \ell \in \mathcal{L}; n \in \mathcal{N} : & p_{jn\ell} = \eta q_{jn\ell} \\
j \in \mathcal{J}; \ell \in \mathcal{L}; n \in \mathcal{N} : & e_{jn\ell} = p_{jn\ell} \pi_{n\ell} \Delta_n \\
j \in \mathcal{J}; \ell \in \mathcal{L}; n \in \mathcal{N} : & \underline{p}_j \leq p_{jn\ell} \leq \bar{p}_j \\
i \in \mathcal{I}; n \in \mathcal{N}\ \{0\} : & s_{in} = s_{in-} + \displaystyle\sum_{j \in \mathcal{J}} A_{ji} \sum_{\ell \in \mathcal{L}} \pi_{n\ell} q_{jn-\ell} \Delta_n + F_{in} \\
i \in \mathcal{I}; n \in \mathcal{N}\ \{0\} : & 0 \leq s_{in} \leq \bar{s}_i \\
i \in \mathcal{I}; n \in \mathcal{N}_T : & s_{in} \geq \bar{g}_i \\
n \in \mathcal{N}\ \{0\} : & c_n = R_{n-} c_{n-} + \displaystyle\sum_{j \in \mathcal{T} \cup \mathcal{P}} \sum_{\ell \in \mathcal{L}} e_{jn-\ell} S_{n-\ell}
\end{array}
$$

Indifference pricing

- Consider M delivery contracts $m \in \mathcal{M} = \{1, ..., M\}$ of individual consumers
  - Different demand levels $D_{mn\ell}$ for different price levels
  - Electricity is delivered at fixed prices K_m (EUR/MWh)
- **Indifference pricing:** Determine the smallest possible price such that the producer meets the obligations from the contracts; new mix of expectation and AVaR γ should not be worse than without the contract

$$\min_{K,q,p,e,c,s,h} \sum_{m \in \mathcal{M}} K_m$$

$$\text{s.t. } (4) - (10)$$

$$\text{AVaR-constraints}$$

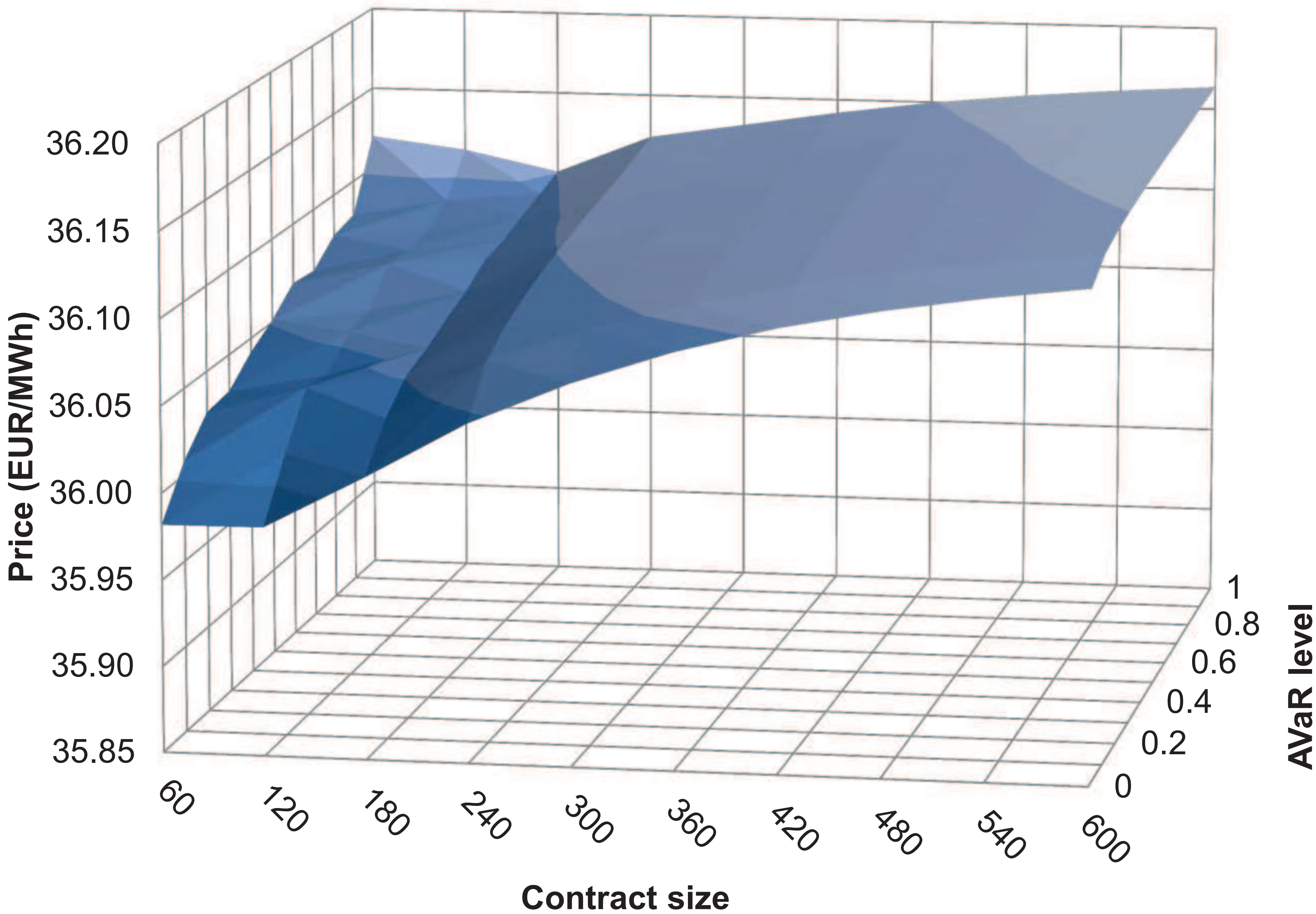
$$\gamma \geq \gamma *$$

$$\sum_{j \in \mathcal{T} \cup \mathcal{P}} e_{jn\ell} = h_{n\ell} + \Delta_n \pi_{n\ell} \sum_{m \in \mathcal{M}} D_{mn\ell}$$

$$c_n = R_{n^-} c_{n^-} + \sum_{\ell \in \mathcal{L}} S_{n\ell} h_{n\ell} + \Delta_n \sum_{\ell \in \mathcal{L}} \pi_{n\ell} \sum_{m \in \mathcal{M}} K_m D_{mn\ell}$$

$$s_{in} \geq s_{in}^* \quad (n \in \mathcal{N}_T)$$

Indifference pricing for a constant load, one contract



Summary

- Optimization of PSHP operations by multistage stochastic programming with aggregation based on price levels
- Model aims at large reservoirs (seasonal storages)
  - Regime-switching model for electricity spot prices
  - Factor model for inflows, taking into account seasonality
- Valuation of power contracts with indifference pricing
- Open:
  - Modelling stochastic load profiles
  - Extension to several contracts (portfolio effects)
  - Comparison with other approaches
  - Scenario generation based on "nested distance" concept
- Further directions of research:
  - Alternative aggregation techniques
  - Inclusion of intraday market