# Stochastic volatility in electricity markets: The impact of wind production on electricity prices

Almut E. D. Veraart

Imperial College London

Statistics in Energy session at the 12th SMSA Workshop
Wroclaw 19–20 February 2015

# The problem

- How can we model electricity day-ahead ("spot") prices?
- What is the impact of wind power production (and other renewable sources) on electricity prices in the European Energy Market?
- Renewable sources are very volatile and weather-dependent.
- Why is it important?
    - Energy markets are of key importance to modern economies.
    - We need reliable stochastic models for spot prices, forward contracts, futures and option prices.
    - Risk management.

# Stylised facts of electricity spot prices

- Equilibrium prices: Supply and demand determine the spot price (results in some form of mean-reversion)
- Non-Gaussian returns
- (Semi-) heavy-tailed distributions
- Volatility clusters and time-varying volatility
- Strong seasonality (over short and long time horizons)
- Extreme spikes used to appear, but they are not that common anymore in the EEX market
- Negative spot prices: Permitted in EEX spot auctions since September 2008. First occurrence: October 2008.

# Data description

- We study electricity prices and wind data for a time period from 01.01.2011 to 31.07.2014 (1308 days).
- Data sources:

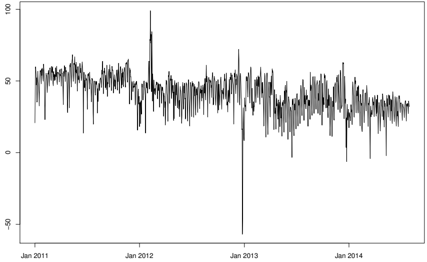
**Prices** Daily EEX Phelix baseload data (for Germany and Austria), downloaded from Datastream and the EPEX spot website.

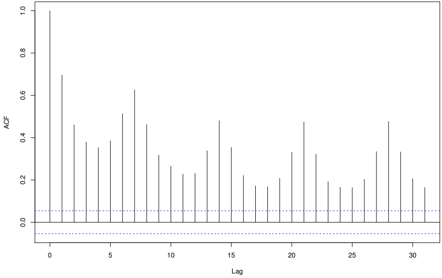
**Volumes** Daily EEX Phelix baseload volume data (for Germany and Austria), downloaded from Datastream and the the EPEX spot website.

**Wind data** We downloaded the *forecasted wind production data* recorded in 15 minute intervals for the four German Transmission System Operator (TSOs) (50hertz, Amprion, Tennet, TransnetBW) and one Austrian TSO (APG). These data have been aggregated to obtain *daily* forecasts for the wind production for each of the five TSOs.

# Daily baseload prices from 01.01.2011 to 31.07.2014

- Summary: Min. -56.87; Median 43.10; Mean 41.96; Max. 98.98.
- Figure 1: Time series plot of the baseload prices.
- Figure 2: Autocorrelation plot of the baseload prices.



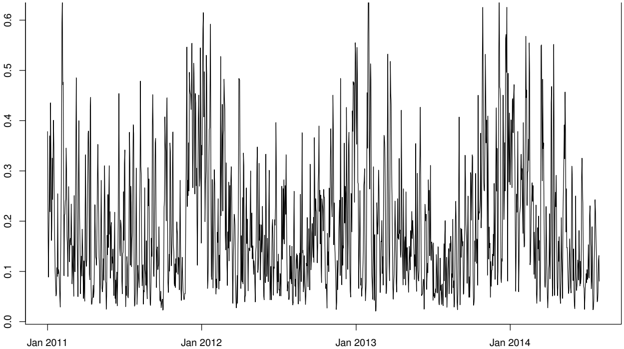



# The predicted wind penetration index

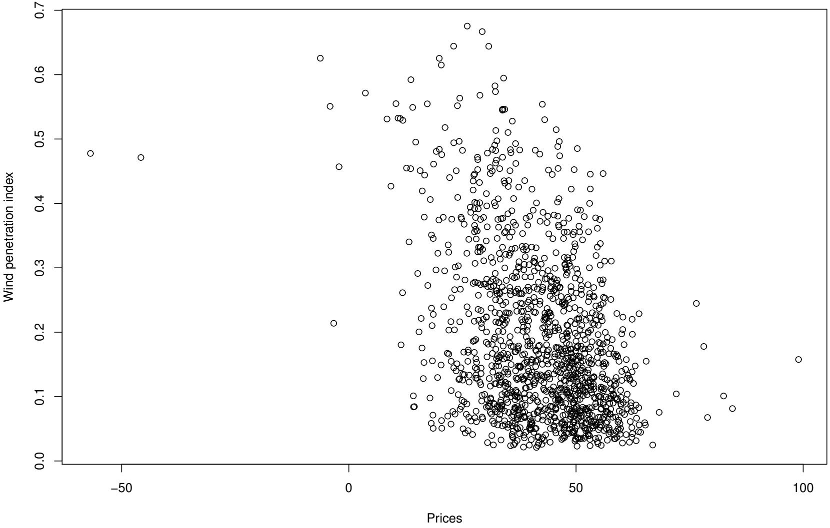
- predicted wind penetration on day *i* = predicted wind feed-in on day *i*/predicted volume of electricity production for day *i*
- Summary:

| Min. | 1st Qu. | Median | Mean | 3rd Qu. | Max. |
|---|---|---|---|---|---|
| 0.02104 | 0.08796 | 0.15480 | 0.19050 | 0.26450 | 0.67540 |

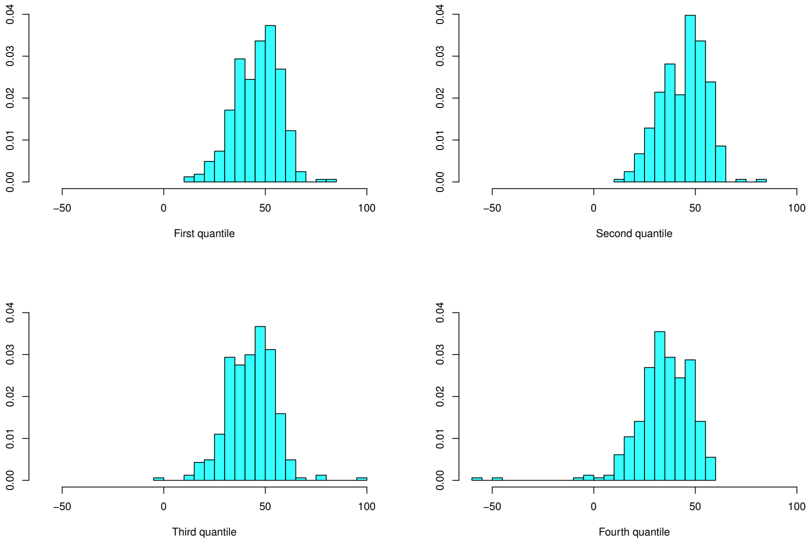
- Time series plot:



# Relation between wind penetration and prices



# Relation between wind penetration and prices cont'd



# How can we incorporate the wind data into the price model?

- The predicted wind penetration index can be viewed as forward-looking information since the information is available before the prices for the next day are determined in the auction market.
- Previous studies have included such information in discrete-time models, such as ARMAX-GARCHX models (see Ketterer (2014, Energy Economics)), where the wind is treated as an exogenous "X" variable.
- We are interested in a continuous-time model. We could consider e.g. CARMA-X or regime-switching models (Cartea, Figueroa, Geman (2009, Applied Mathematical Finance))

# Lévy semistationary ($\mathcal{LSS}$) processes

- Barndorff-Nielsen, Benth, Veraart (2013, Bernoulli) found that Lévy semistationary (LSS) processes can describe energy spot prices very well.

# Lévy semistationary ($\mathcal{LSS}$) processes

- Barndorff-Nielsen, Benth, Veraart (2013, Bernoulli) found that Lévy semistationary (LSS) processes can describe energy spot prices very well.
- Today we are going to look at an extension of this class of models which allows to incorporate forward-looking information such as the predicted wind penetration index.

# Lévy semistationary ($\mathcal{LSS}$) processes

- Barndorff-Nielsen, Benth, Veraart (2013, Bernoulli) found that Lévy semistationary (LSS) processes can describe energy spot prices very well.
- Today we are going to look at an extension of this class of models which allows to incorporate forward-looking information such as the predicted wind penetration index.
- An LSS process $Y = \{Y(t)\}_{t \in \mathbb{R}}$ on $\mathbb{R}$ is defined as

$$Y(t) = \int_{-\infty}^{t} g(t-s)\sigma(s-)dL(s), \tag{1}$$

where $L$ denotes a Lévy process; $g : \mathbb{R} \to \mathbb{R}$ denotes a deterministic, weight function satisfying $g(s) = 0$ whenever $s < 0$; $\sigma$ denotes a càdlàg, adapted stochastic volatility process. Assume independence of $\sigma$ and $L$. Assume the suitable integrability conditions hold.

# CARMA(p,q) process

- CARMA(p,q) for $p > q$.
- AR and MA polynomials:

$$P^{\mathrm{AR}(p)}(z) = z^p + a_1 z^{p-1} + \cdots + a_p,$$
$$P^{\mathrm{MA}(q)}(z) = b_0 + b_1 z + \cdots + b_{p-1} z^{p-1},$$

$b_q = 1$ and $b_j = 0$ for $q < j < p$.

- Assumption: No common roots.
- Write formally

$$P^{\mathrm{AR}(p)}(D)Y(t) = P^{\mathrm{MA}(q)}(D)D\widetilde{L}(t),$$

where $D = \frac{d}{dt}$, $L$ Lévy process.

# Interpretation in terms of a state space representation

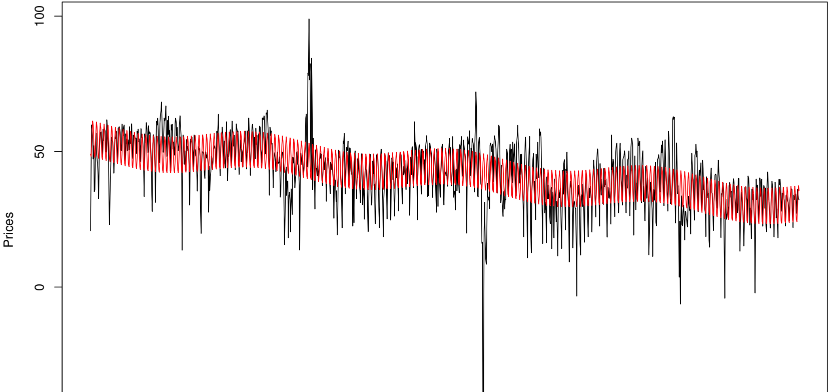
- $Y(t) = \mathbf{b}^\top \mathbf{V}(t)$, $\quad d\mathbf{V}(t) = \mathbf{A}\mathbf{V}(t)dt + \mathbf{e}d\widetilde{L}(t)$, where

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & 1 \\ -a_p & -a_{p-1} & \cdots & \cdots & -a_1 \end{pmatrix}, \quad \mathbf{e} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} b_0 \\ b_1 \\ \vdots \\ b_{p-1} \end{pmatrix}.$$

- Assume: All eigenvalues of $\mathbf{A}$ have negative real parts.
- Then $\mathbf{V}(t) = \int_{-\infty}^{t} e^{\mathbf{A}(t-s)}\mathbf{e}\, d\widetilde{L}(s)$ is the (strictly) stationary solution of the SDE above.
- Choose $g(x) = \mathbf{b}^\top e^{\mathbf{A}x}\mathbf{e}$.

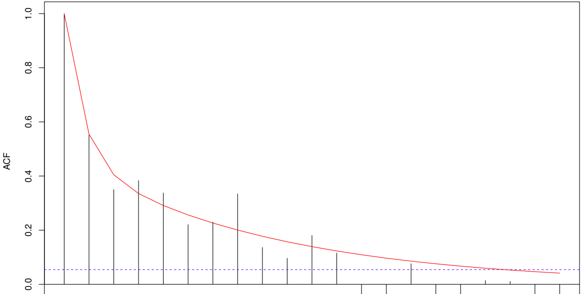
# Arithmetic model

- Define the *arithmetic* spot price vector, $S = (S(t))_{t \geq 0}$ by
$$S(t) = D(t) + Y(t).$$
  - $D$ is a seasonality and trend function and
  - $Y$ is an LSS process.



# Model estimation

1. We fit a CARMA(2,1) process and estimate the CARMA(2,1) parameters in the kernel function $g$ by quasi–maximum likelihood estimation.
2. In a next step, we recover the observations of the underlying Lévy process $\widetilde{L}_i$ using the algorithm developed by Brockwell, Davis & Yang (2011).
3. Finally, we extend the model beyond the Lévy framework.



# Model extension: A regime-switching LSS process

- The original LSS model has the structure

$$Y(t) = \int_{-\infty}^{t} g(t-s)dM(s), \quad \text{for } dM(s) = \sigma(s-)dL(s).$$

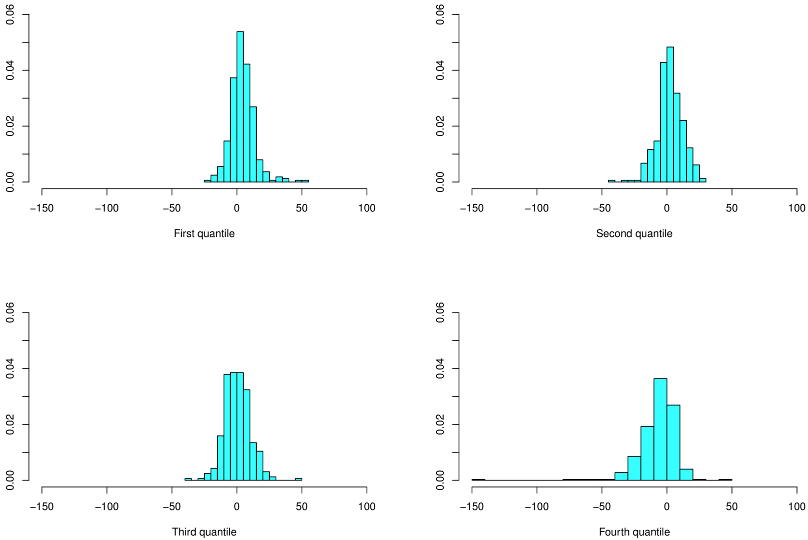
- We extend the model by introducing regime-switching:
- We introduce the (forward-looking) variable of the predicted wind penetration index $\rho$, where

$$\rho(s) = \begin{cases} 1, & \text{if the predicted wind penetration at time } s \text{ is "high"} \\ 0, & \text{if the predicted wind penetration at time } s \text{ is "low".} \end{cases}$$

- Then set

$$dM(s) = \rho(s)dM^{(1)}(s) + (1-\rho(s))dM^{(2)}(s).$$

# Selecting the regime switch based on quantile information

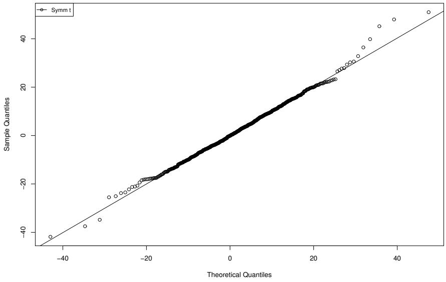


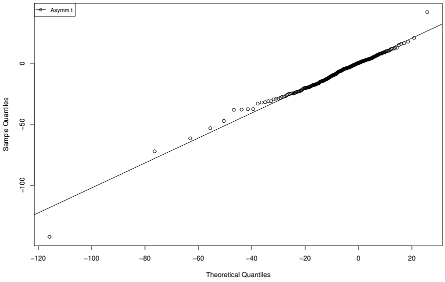
# Low and high regime

- In our data the cut-off between the low and the high regime is chosen to be at the level of 26% of the predicted wind penetration index.
- We found that the generalised hyperbolic distribution fits the data well.
- For the low regime, the symmetric Student-t distribution is the preferred model
- For the high regime, the asymmetric Student-t distribution is the preferred model
- Parameter estimates:

| | $\nu$ | $\mu$ | $\sigma$ | $\gamma$ |
|---|---|---|---|---|
| Low | 6.70 | 2.26 | 9.77 | 0 |
| High | 4.76 | 0.57 | 11.31 | -7.68 |

# Diagnostic plots



# The full model

- The spot price is modelled as $S(t) = D(t) + Y(t)$, where
$$Y(t) = \int_{-\infty}^{t} g(t-s)dM(s).$$
- Here
$$dM(s) = \rho(s)dM^{(1)}(s) + (1-\rho(s))dM^{(2)}(s),$$
where
$$dM^{(1)}(s) = \left(\mu^{(1)} + \gamma\left(\sigma^{(i)}(s)\right)^2\right)ds + \sigma^{(1)}(s)dW^{(1)}(s),$$
$$dM^{(2)}(s) = \mu^{(2)}ds + \sigma^{(2)}(s)dW^{(2)}(s),$$
for independent Brownian motions $W^{(1)}$ and $W^{(2)}$.
- The stochastic volatility processes $\sigma^{(i)}$ are chosen as Ornstein-Uhlenbeck processes with inverse Gamma marginal distribution.

# Summary and Outlook

- We found that increasing wind production tends to decrease electricity prices, but also increases their volatility.
- An extension of the class of Lévy stationary processes has been proposed to address this issue leading to a regime-switching Lévy semistationary process.
- In future research it will be interesting to also derive a model for the wind production itself.

# Summary and Outlook

- We found that increasing wind production tends to decrease electricity prices, but also increases their volatility.
- An extension of the class of Lévy stationary processes has been proposed to address this issue leading to a regime-switching Lévy semistationary process.
- In future research it will be interesting to also derive a model for the wind production itself.

Thank you.