Robust estimation and forecasting of the long-term seasonal component (LTSC) of electricity spot prices

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Introduction

When building electricity spot price models we should address two questions:

- How to estimate the trend-seasonal component?
- How to forecast it?
3 approaches to LTSC modeling

1. Piecewise constant functions or dummies
   - Non-smooth LTSC with jumps between months

2. Sinusoidal functions (also coupled with EWMA)
   - Annual periodicity can hardly be observed in market data

3. Wavelets or other nonparametric smoothers
   - More robust to outliers and less periodic
   - ... but forecasting of a nonparametric LTSC is not trivial
3 LTSC fits to Nord Pool spot prices

Robust estimation and forecasting of the LTSC
3 stochastic components (residuals)

Wavelet-based residuals

Sine-based residuals

Dummies-based residuals

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Robust estimation and forecasting of the LTSC
3 MRJD fits: $dX = (\alpha - \beta X) dt + \sigma dB + \mathcal{N}(\mu, \gamma) dN(\lambda)$

- $\alpha = 42.42$, $\beta = 0.29$, $(\alpha/\beta = 146.78)$, $\sigma = 11.69$, $\mu = 24.85$, $\gamma = 121.05$, $\lambda = 0.04$
- $\alpha = 24.97$, $\beta = 0.17$, $(\alpha/\beta = 143.99)$, $\sigma = 11.36$, $\mu = 19.48$, $\gamma = 109.87$, $\lambda = 0.06$
- $\alpha = 5.85$, $\beta = 0.05$, $(\alpha/\beta = 128.25)$, $\sigma = 11.25$, $\mu = 15.41$, $\gamma = 106.69$, $\lambda = 0.07$
Conclusions ... so far

- In-sample the wavelet-based LTSC (red) is clearly the best
  - This is not a surprise, given the more degrees of freedom the nonparametric models have
- But how does the wavelet-based LTSC perform out-of-sample?
- How to forecast it?
Agenda

- Introduction
- **Datasets and models**
- Estimating and forecasting the LTSC
- Results
- Conclusions

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Robust estimation and forecasting of the LTSC
6 markets:
- NSW, Australia (2038 obs.)
- EEX, Germany (3754 obs.)
- Nord Pool, Scandinavia (3240 obs.)
- ISO-NE, U.S. (3770 obs.)
- NYISO, U.S. (2588 obs.)
- PJM, U.S. (1944 obs.)
304 models: Simple and sine-based models

- Simple models (1**000**) → 16 models
  - mean, median, linear regression
  - linear/exponential decay from the current spot price to the median
  - dummies: mean-based, median-based

- Sines fitted to raw prices (2***00) → 24 models
  - 1-4 sines used
  - periods estimated or set to $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ of a year

- Sines fitted to spike-filtered prices (3****0) → 48 models
  - Spikes replaced by the mean or the upper/lower 2.5% quantiles of the deseasonalized prices
304 models: Wavelet-based models

- Wavelets with an exponential decay to the median fitted to raw prices ($4^{***0*}$) → 48 models
  - 4 types of wavelets (Daubechies, Coiflets)
  - 3 approximation levels (6, 7, 8)
  - 2 exponential decay constants

- Wavelets with a linear decay to the median fitted to raw prices ($5^{***00}$) → 24 models

- Wavelets with an exponential decay to the median fitted to spike-filtered prices ($6^{*****}$) → 96 models
  - Spikes replaced by the mean or the upper/lower 2.5% quantiles of the deseasonalized prices

- Wavelets with an exponential decay to the median fitted to spike-filtered prices ($7^{****0}$) → 48 models
Agenda

- Introduction
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- **Estimating and forecasting the LTSC**
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2 calibration windows (rolling windows):
- 2-year (730 days)
- 3-year (1095 days)

6 forecast horizons:
- 1-7 day, 8-30, 31-90
- 91-182 (2nd Qtr)
- 183-274 (3rd Qtr)
- 275-365 (4th Qtr)
Estimation: Dummies and sines

Robust estimation and forecasting of the LTSC
Forecasting: Dummies and sines

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Robust estimation and forecasting of the LTSC
Wavelets

Decomposition of a signal

Original signal

Approximation 1 level

Approximation 7 level

Details 1 level

Details 7 level

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Robust estimation and forecasting of the LTSC
Wavelets

Decomposition of a signal

Original signal

Approximation 1 level

Details 1 level

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Robust estimation and forecasting of the LTSC
Decomposition of a signal

Original signal

Approximation 1 level

Approximation 7 level

Details 1 level

Details 7 level

Robust estimation and forecasting of the LTSC
Estimation and forecasting: Wavelets cont.

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Robust estimation and forecasting of the LTSC
Estimation and forecasting: Wavelets cont.
Agenda

- Introduction
- Datasets and models
- Estimating and forecasting the LTSC
- Results
- Conclusions
For every dataset $d_i$ and every forecasting horizon $h_j$ we **rank** the models according to MAE, MSE and MAPE.
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For each dataset we calculate the geometric means $GM(MAE^*, d)$ and $GM(MSE^*, d)$ of the ranks.
For every dataset $d_i$ and every forecasting horizon $h_j$ we rank the models according to MAE, MSE and MAPE.

- For each dataset we calculate the geometric means $GM(MAE_{*,d})$ and $GM(MSE_{*,d})$ of the ranks.
- For each horizon we calculate the geometric means $GM(MAE_{h,*})$ and $GM(MSE_{h,*})$ of the ranks.
Evaluating forecasting performance

For every dataset $d_i$ and every forecasting horizon $h_j$ we rank the models according to MAE, MSE and MAPE.

- For each dataset we calculate the geometric means $GM(\text{MAE}_{i,*}, d)$ and $GM(\text{MSE}_{i,*}, d)$ of the ranks.
- For each horizon we calculate the geometric means $GM(\text{MAE}_{h,*})$ and $GM(\text{MSE}_{h,*})$ of the ranks.
- We also calculate the global geometric means $GM(\text{MAE}_{*,*})$ and $GM(\text{MSE}_{*,*})$ of the ranks.
Evaluating forecasting performance

For every dataset $d_i$ and every forecasting horizon $h_j$ we rank the models according to MAE, MSE and MAPE.

For each dataset we calculate the geometric means $\text{GM}(\text{MAE}_{*,d})$ and $\text{GM}(\text{MSE}_{*,d})$ of the ranks.

For each horizon we calculate the geometric means $\text{GM}(\text{MAE}_{h,*})$ and $\text{GM}(\text{MSE}_{h,*})$ of the ranks.

We also calculate the global geometric means $\text{GM}(\text{MAE}_{*,*})$ and $\text{GM}(\text{MSE}_{*,*})$ of the ranks.

Finally, we calculate $\text{MAPE}_{*,d}$, $\text{MAPE}_{h,*}$ and the global $\text{MAPE}_{*,*}$. 
Results

Top 15 models according to each of the three global forecast error measures

<table>
<thead>
<tr>
<th>No.</th>
<th>GM(MAE*,*)</th>
<th>Model</th>
<th>GM(MSE*,*)</th>
<th>Model</th>
<th>MAPE*,*</th>
<th>Model</th>
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<tbody>
<tr>
<td>1</td>
<td>17.13</td>
<td>731310</td>
<td>10.84</td>
<td>623322</td>
<td>30.04%</td>
<td>734110</td>
</tr>
<tr>
<td>2</td>
<td>23.37</td>
<td>723310</td>
<td>13.75</td>
<td>622322</td>
<td>30.04%</td>
<td>732110</td>
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<tr>
<td>3</td>
<td>23.93</td>
<td>631312</td>
<td>14.71</td>
<td>624322</td>
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<td>4</td>
<td>24.86</td>
<td>623322</td>
<td>20.98</td>
<td>631322</td>
<td>30.06%</td>
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<tr>
<td>5</td>
<td>24.86</td>
<td>722110</td>
<td>24.82</td>
<td>631312</td>
<td>30.08%</td>
<td>731110</td>
</tr>
<tr>
<td>6</td>
<td>25.16</td>
<td>723320</td>
<td>24.91</td>
<td>633122</td>
<td>30.09%</td>
<td>724310</td>
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<tr>
<td>7</td>
<td>25.58</td>
<td>721110</td>
<td>25.63</td>
<td>624122</td>
<td>30.14%</td>
<td>731310</td>
</tr>
<tr>
<td>8</td>
<td>25.91</td>
<td>724310</td>
<td>25.82</td>
<td>621322</td>
<td>30.15%</td>
<td>623322</td>
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<tr>
<td>9</td>
<td>26.44</td>
<td>623312</td>
<td>28.87</td>
<td>634122</td>
<td>30.16%</td>
<td>624322</td>
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<tr>
<td>10</td>
<td>26.97</td>
<td>724110</td>
<td>29.50</td>
<td>621122</td>
<td>30.18%</td>
<td>722310</td>
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<td>11</td>
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<tr>
<td>12</td>
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<tr>
<td>13</td>
<td>29.94</td>
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<td>30.20%</td>
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<tr>
<td>14</td>
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<td>32.77</td>
<td>422302</td>
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<tr>
<td>15</td>
<td>31.25</td>
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<td>32.88</td>
<td>424302</td>
<td>30.26%</td>
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</table>
... models according to each of the three global forecast error measures

<table>
<thead>
<tr>
<th>No.</th>
<th>GM(MAE*,*) Model</th>
<th>GM(MSE*,*) Model</th>
<th>MAPE*,* Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>51</td>
<td>.</td>
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<td>30.51% 423302</td>
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<td>53.42 523300</td>
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<td>71.88 130001</td>
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<tr>
<td>70</td>
<td>62.27 423302</td>
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<tr>
<td>79</td>
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<td>77.29 524200</td>
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<tr>
<td>82</td>
<td>70.87 120005</td>
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<tr>
<td>105</td>
<td>.</td>
<td>.</td>
<td>30.91% 524300</td>
</tr>
<tr>
<td>120</td>
<td>.</td>
<td>.</td>
<td>31.14% 130005</td>
</tr>
<tr>
<td>128</td>
<td>98.93 120008</td>
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<td>173</td>
<td>.</td>
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<tr>
<td>182</td>
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<td>148.64 130008</td>
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</tr>
<tr>
<td>209</td>
<td>192.24 324320</td>
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<tr>
<td>225</td>
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<td>34.47% 130008</td>
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<tr>
<td>226</td>
<td>206.55 232200</td>
<td>.</td>
<td>.</td>
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<tr>
<td>228</td>
<td>.</td>
<td>.</td>
<td>36.91% 331110</td>
</tr>
<tr>
<td>241</td>
<td>.</td>
<td>.</td>
<td>37.43% 231300</td>
</tr>
</tbody>
</table>
The number of times models from a given family are ranked in the top 5, 20 and 50 of all 304 models according to $GM(MAE_{h,*})$, $GM(MSE_{h,*})$ and $MAPE_{h,*}$ for each of the six forecast horizons $h = 1,\ldots,6$. 

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Conclusions

- A comprehensive study on the forecasting of the LTSC
- Over 300 models examined, including commonly used and new approaches
- Wavelet-based models outperform sine-based and monthly dummy models
  - Both in-sample (modeling) and out-of-sample (forecasting)
- Validity of stochastic models built on sines or monthly dummies is questionable