Recent trends and advances in electricity price forecasting (EPF)

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http://www.ioz.pwr.wroc.pl/pracownicy/weron/
Agenda

1. Beyond point forecasts ⇒ probabilistic forecasts
2. Combining forecasts
   - Point forecasts
   - Probabilistic forecasts
3. Variable selection and shrinkage
   - LASSO
   - Elastic nets
4. Guidelines for evaluating forecasts
Beyond point forecasts

- Variability of supply and demand has become a challenge to the utility industry in the smart grid era (Hong & Fan, 2016, IJF)
- Resulting extreme variability of electricity prices
  - In the day-ahead market
  - Even more so in the intraday market
- Probabilistic (interval, density) forecasting has a lot to offer (Nowotarski & Weron, 2016, RePEc)
  - Useful in practice for risk management and decision-making
- GEFCom2012 (point) ⇒ GEFCom2014 (probabilistic forecasts)
Probabilistic (interval, density) forecasting


- Improved assessment of future uncertainty
- Ability to plan different strategies for the range of possible outcomes
- Possibility of more thorough forecast comparisons
1. Beyond point forecasts

GEFCom2014 Global Energy Forecasting Competition 2014
(Hong, Pinson, Fan et al., 2016, IJF)

- Incremental data sets released on weekly basis
- Price Track:
  - 287 contestants
  - Submit 99 quantiles for 24h load periods of the next day
Price Track

Task 10

Task 11

Task 12

Task 13

Task 14

Task 15

Real price
Benchmark
Median
25% and 75% quantiles
5% and 95% quantiles

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Price Track: Top winning teams
(1st and) 2nd place for QRA!

1. Pierre Gaillard, Yannig Goude, Raphaël Nedellec (EDF R&D, F)
2. Katarzyna Maciejowska, Jakub Nowotarski (Wrocław UT, PL)
3. Grzegorz Dudek (Częstochowa UT, PL)
4. Zico Kolter, Romain Juban, Henrik Ohlsson, Mehdi Maasoumy (C3 Energy, USA)
5. Frank Lemke (KnowledgeMiner Software, D)
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Point forecast averaging: The idea

Dates back to the 1960s and the works of Bates, Crane, Crotty & Granger

‘AI world’: committee machines, ensemble averaging, expert aggregation

\[ f_c = \sum_{i=1}^{N} w_i f_i \]
Interval forecast averaging

- For point forecasts: \( f_c = \sum_{i=1}^{N} w_i f_i \)
  (e.g. a linear regression model)

- For interval forecasts the above formula does not hold

- A linear combination of \( q \)-th quantiles is not the \( q \)-th quantile of a linear combination of random variables

\[
x^q_c \neq \sum_{i=1}^{N} w_i x^q_i
\]

\( \Rightarrow \) Need for development of new approaches
Quantile Regression Averaging (QRA) defined

Computing electricity spot price prediction intervals using quantile regression and forecast averaging

Jakub Nowotarski · Rafał Weron

Abstract We examine possible accuracy gains from forecast averaging in the context of interval forecasts of electricity spot prices. First, we test whether constructing empirical prediction intervals (PI) from combined electricity spot price forecasts leads to better forecasts than those obtained from individual methods. Next, we propose a new method for constructing PI—Quantile Regression Averaging (QRA)—which utilizes the concept of quantile regression and a pool of point forecasts of individual (i.e. not combined) models. While the empirical PI from combined forecasts do not provide significant gains, the QRA-based PI are found to be more accurate than those of the best individual model—the smoothed nonparametric autoregressive model.
Quantile Regression Averaging: The idea

Quantile regression:
Individual point forecasts
Combined interval forecast (e.g. for \( q = 0.05 \) & 0.95)

\[
\min_{\beta_q} \left[ \sum_t \left( q - 1_{y_t < X_t \beta_q} \right) (y_t - X_t \beta_q) \right]
\]

\[ X_t = [1, \hat{y}_{1,t}, \ldots, \hat{y}_{m,t}] \]
\[ \beta_q - \text{vector of parameters} \]

Combined interval forecast (e.g. for \( q=0.05 \) & 0.95)
Quantile regression
Quantile regression

- Linear regression
- Quantile regression, $\alpha=0.95$, $\alpha=0.05$

Interval forecast
How does the score function look like?

For vector $X_t = [1, \hat{y}_{1,t}, \ldots, \hat{y}_{m,t}]$ of point forecasts, i.e. explanatory variables, weights $\beta_q$ are estimated by minimizing:

$$
\min_{\beta_q} \left[ \sum_{\{t: y_t \geq X_t \beta_q\}} q |y_t - X_t \beta_q| + \sum_{\{t: y_t < X_t \beta_q\}} (1 - q) |y_t - X_t \beta_q| \right]
$$
Merging quantile regression with forecast averaging to obtain more accurate interval forecasts of Nord Pool spot prices

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Abstract—We evaluate a recently proposed method for constructing prediction intervals, which utilizes the concept of quantile regression (QR) and a pool of point forecasts of different time series models. We find that in terms of interval forecasting of Nord Pool day-ahead prices the new QR-based approach significantly outperforms prediction intervals obtained from standard, as well as, semi-parametric autoregressive time series models.

...
QRA at work

- Nord Pool hourly prices (2012-2013)
  - **Seven** months for calibration of individual models
  - **Four** weeks for calibration of quantile regression
  - **26** weeks for evaluation of interval forecasts
- **Six** individual point forecasting models
  - AR, TAR, SNAR, MRJD, NAR, FM
Evaluation of forecasts

- 50% and 90% two-sided day-ahead prediction intervals
- Two benchmark models: AR and SNAR
- Christoffersen’s (1998, IER) test for unconditional and conditional coverage

The focus on the sequence: 

\[ I_t = \begin{cases} 
1 & y_t \in [\hat{y}_t^L, \hat{y}_t^U] \\
0 & y_t \notin [\hat{y}_t^L, \hat{y}_t^U] 
\end{cases} \]

- **Conditional Coverage test**
  - (UC + independence)
  - Asymptotically \( \chi^2(2) \)

- **Unconditional Coverage test**
  - Asymptotically \( \chi^2(1) \)
## Results: Unconditional coverage

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*Unconditional coverage*

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*Mean width (STD of interval width)*
Results: Christoffersen’s test

2. Combining forecasts

QRA Case Study

Conditional coverage LR

Unconditional coverage LR

AR

SNAR

QRA

Hour

50% PI 90% PI

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Recent trends in EPF

3.3.2017, 4th EPM&F Forum

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FQRA: When the number of predictors is large

(Maciejowska, Nowotarski & Weron, 2016, IJF)
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Automated variable selection

Consider a general regression:

$$\hat{y}_i = \sum_{j=1}^{p} \beta_j x_{i,j} + \varepsilon_i$$

How to select predictors $x_{i,j}$? How to estimate $\beta_j$’s?

- Single-step elimination of insignificant predictors
  - In EPF: Gianfreda & Grossi (2012)
- Stepwise regression
  - Forward stepwise selection
  - Backward stepwise elimination
  - In EPF: Karakatsani & Bunn (2008), Misiorek (2008), Bessec et al. (2016), Keles et al. (2016)
What is shrinkage (regularization)?

- Minimize the residual sum of squares (RSS) + a penalty function of the betas:

\[
\hat{\beta} = \arg\min_{\beta_j} \left\{ \sum_{i=1}^{N} \left( y_i - \sum_{j=1}^{p} \beta_j x_{i,j} \right)^2 + \lambda \sum_{j=1}^{n} |\beta_j|^q \right\}
\]

- Ridge regression \((q = 2)\)
  - Introduced by: Hoerl & Kennard (1970, Technometrics)
  - In EPF: Barnes & Balda (2013)

- Least Absolute Shrinkage & Selection Operator (LASSO; \(q = 1\))
  - Introduced by: Tibshirani (1996, JRSSB)
How does it work?

Lasso

Ridge regression

Blue areas – constraint regions, i.e., $|\beta_1| + |\beta_2| \leq t$ and $\beta_1^2 + \beta_2^2 \leq t$

Red ellipses – contours of the least squares error function
Elastic net

- RSS penalized by a mixed quadratic and linear shrinkage factor

\[
\hat{\beta}^{\text{EN}} = \arg\min_{\beta_j} \left\{ \text{RSS} + \lambda \left( \frac{1 - \alpha}{2} \sum_{j=1}^{n} \beta_j^2 + \alpha \sum_{j=1}^{n} |\beta_j| \right) \right\}
\]

- Introduced by: Zou & Hastie (2015, JRSSB)
- In EPF: Uniejewski, Nowotarski & Weron (2016, Energies)
How $\hat{\beta}$’s change when $\lambda$ increases?

- **Left**: Ridge regression with $\lambda \in (0, 2000)$, linear scale
- **Center**: Elastic net with $\alpha = 0.5$ and $\lambda \in (0, 1)$, log-scale
- **Right**: Lasso with $\lambda \in (0, 1)$, log-scale

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3. Variable selection and shrinkage

Results: WMAE errors
(Uniejewski et al., 2016, Energies)

Full model: fARX, fAR

\[
\hat{p}_{d,h} = \sum_{i=1}^{24} (\beta_{h,i} p_{d-1,i} + \beta_{h,i+24} p_{d-2,i} + \beta_{h,i+48} p_{d-3,i}) + \beta_{h,73} p_{d-7,h}
\]
72 hourly prices from the three previous days

+ \sum_{j=1}^{3} (\beta_{h,j+73} p_{d-j}^{\text{min}} + \beta_{h,j+76} p_{d-j}^{\text{max}} + \beta_{h,j+79} p_{d-j})
minimum, maximum & average price of the three previous days

+ \beta_{h,83} z_{d,h} + \beta_{h,84} z_{d-1,h} + \beta_{h,85} z_{d-7,h} + \beta_{h,86} y_{d,h}
exogenous variables

+ \sum_{k=1}^{7} \beta_{h,86+k} D_k + \sum_{k=1}^{7} \beta_{h,93+k} D_k z_{d,h} + \sum_{k=1}^{7} \beta_{h,100+k} D_k p_{d-1,h}
weekly seasonality

+ \varepsilon_{d,h}

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Full ARX model

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Selection and shrinkage methods

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### Variable significance across hours

(Ziel & Weron, 2016, RePEc)

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[Table of data: Mean occurrence (in %) of the multivariate lasso model parameters across all 12 datasets and the full out-of-sample data (see Table 5). A heat map is used to indicate more (in green) and less (in red) commonly-selected variables. Continued on next page.]
Variable significance across hours cont.

(Rziel & Weron, 2016, RePec)
Agenda

1. Beyond point forecasts ⇒ probabilistic forecasts
2. Combining forecasts
   - Point forecasts
   - Probabilistic forecasts
3. Variable selection and shrinkage
   - LASSO
   - Elastic nets
4. Guidelines for evaluating forecasts
Maximizing *sharpness* subject to *reliability*  
(Gneiting & Katzfuss, 2014; Nowotarski & Weron, 2016)

- **Reliability** refers to statistical consistency ($x\%$ PI covers $x\%$ of obs.)
- **Sharpness** refers to how tightly the PI covers the observations

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<td><strong>Sharpness (and reliability)</strong></td>
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Take-home messages

1. Beyond point forecasts  ⇒  probabilistic forecasts
2. Combining forecasts
   - Point forecasts
   - Probabilistic forecasts
3. Variable selection and shrinkage
   - LASSO
   - Elastic nets
4. Guidelines for evaluating forecasts