Associative learning using Ising-like model

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Abstract. In this paper, a new computational model of associative learning is proposed, which is based on the Ising model. Application of the stochastic gradient descent algorithm to the proposed model yields an on-line learning rule. Next, it is shown that the obtained new learning rule generalizes two well-known learning rules, i.e., the Hebbian rule and the Oja's rule. Later, the fashion of incorporating the cognitive account into the obtained associative learning rule is proposed. At the end of the paper, experiments were carried out for testing the backward blocking and the reduced overshadowing and blocking phenomena. The obtained results are discussed and conclusions are drawn.

Keywords: associative learning, Ising model, energy-based model, Hebbian rule, Oja's rule, Rescorla-Wagner model, backward blocking, reduced overshadowing

1 Introduction

Understanding animal or human learning process remains the most intriguing question in psychology, artificial intelligence, and cognitive science [13, 19]. There are different approaches to understand learning processes. According to the Marr's tri-level hypothesis [11], the learning process can be considered at the representational level, i.e., by studying neurobiological properties of the brain [2], or at the algorithmic level, for example, by investigating how many information humans can process [6], or at the computational level which aims at describing what information are processed and what abstract representation has to be used in order to solve the learning problem [22]. In this work, we focus on modelling learning process at the computational level.

In the literature, two prominent types of accounts have been offered to explain the learning process phenomenon at the computational level [19, 20], namely, associative (contingency) perspective, and cognitive (inferential, propositional) perspective. The first one tries to explain learning process as an unconscious process in which associations are built between cues (stimuli) and outcome (target) [5, 18]. There are different models which are based on the associative account, e.g., classical Hebbian rule [7] and Rescorla-Wagner model [18], and more recent probabilistic models which use Kalman filter [5, 9] or noisy logic gate [9]. In the second approach it is claimed that an explicit reasoning process leads to inferences about causal relations between stimuli and target [15]. Recently, there were a series of probabilistic models in which the human inference is explained in terms of Bayesian learning paradigm [22].

In this paper, we concentrate on the associative account (an unaware learning phase) and try to combine it with the cognitive perspective (an aware learning phase). Such approach seems to be a reasonable direction because recent studies show that both types of learning processes co-exist [20]. Our model of associative learning is based on the Ising model [4, 12]. The Ising model, originally developed in the statistical physics, is an energy-based model which associates an energy (a real value) with a system's state. This model has been successfully applied to many real-life problems, e.g., image de-noising [3], opinion evolution in closed community (also known as Sznajd model) [21], or biophysical dynamics modelling [10]. Moreover, the Ising model is a starting point in many models used in the field of machine learning, e.g., Hopfield networks [8] or Boltzmann machines [1].

The contribution of the paper is threefold:

1. A new computational model for associative learning basing on the Ising model is proposed. The model leads to an on-line learning rule which generalizes the well-known Hebbian rule and Oja's rule.
2. The proposed approach is qualitatively evaluated on benchmark experiments, namely, *backward blocking* and *reduced overshadowing and blocking*.
3. A fashion of incorporating cognitive perspective into the associative learning rule is outlined.

The paper is organized as follows. In Section 2 first the problem of associative learning is stated, and then the classical Rescorla-Wagner model is given (Section 2.1), and the proposed model is described (Section 2.2). Additionally, the generalizations of the well-known learning rules are derived. In Section 3 the experiments are carried out: backward blocking (Section 3.1), and reduced overshadowing and blocking (Section 3.2). Next, the results are discussed in Section 3.3. At the end, the conclusions are drawn in Section 4.

2 Associative learning

The aim of a learning process is to find a dependency between cues (stimuli) and outcomes (responses or targets). The associative account for learning focuses on analysing links strengthening between stimuli and responses. In other words, it is the study of how animals or humans learn predictive relationships.

We distinguish a vector of D cues, $\mathbf{x} \in \{0,1\}^D$, and a single outcome, $t \in \{-1,1\}$.[1] If the d^{th} cue is present, we write $x_d = 1$, and $x_d = 0$ – otherwise. Additionally, we denote learner's knowledge about the associative strength between d^{th} cue and the outcome as w_d. Hence, we have a vector of D associative weights, $\mathbf{w} \in \mathbb{R}^D$.

[1] In this paper, we decided on coding the outcome using -1 and $+1$ because we want to differentiate the negative and positive responses, respectively.

The goal of the associative learning is to determine the weights values for given N observations, $\mathcal{D} = \{(\mathbf{x}_n, t_n)\}_{n=1}^{N}$. Positive values of weights represent strong relationship while negative values of weights – weak or none influence of cues on the outcome.

In further considerations we assume a model in which the outcome is a weighted sum of the cue activations:

$$y(\mathbf{x}; \mathbf{w}) = \mathbf{w}^{\top}\mathbf{x}. \tag{1}$$

The meaning of the model is straightforward – each cue contributes to the outcome with its associative strength.

2.1 Classical approach – Rescorla-Wagner model

In the classical approach to the associative learning one aims at minimizing the error between the true target value and the predicted one, which is a function in the following form:

$$\mathcal{E}(\mathbf{w}) = \sum_{n=1}^{N} \mathcal{E}_n(\mathbf{w}), \tag{2}$$

where

$$\mathcal{E}_n(\mathbf{w}) = \big(t_n - y(\mathbf{x}_n; \mathbf{w})\big)^2.$$

In general, the stochastic (on-line) gradient descent (SGD) takes the following form ($\eta > 0$ – learning parameter, $\nabla_{\mathbf{w}}$ – gradient operator over $\mathbf{w}$):

$$\mathbf{w} := \mathbf{w} - \eta \nabla_{\mathbf{w}} \mathcal{E}_n(\mathbf{w}), \tag{3}$$

Applying SGD to the error function (2) yields the following learning rule:

$$\mathbf{w} := \mathbf{w} + \eta(t_n - \mathbf{w}^{\top}\mathbf{x}_n)\mathbf{x}_n, \tag{4}$$

which is known as *Rescorla-Wagner model* in the psychology domain [14, 18].

2.2 Our approach – Ising-like model

The energy function In this paper, we propose a model which is driven by other motivation than simple error minimization approach. We want to associate an *energy* (a real value) with current learner's knowledge (state), i.e., weights' values, by proposing *an energy function*. We formulate the energy function as follows.

We know that there must be a strong correlation between the true outcome t and the model $y(\mathbf{x}; \mathbf{w})$. This dependency can be captured by the form $-c\mathbf{w}^{\top}\mathbf{x}t$, where $c > 0$. This has the desired effect of giving lower energy if $y(\mathbf{x}; \mathbf{w})$ and t have the same signs and higher energy – otherwise.

Additionally, we want to favour neighbouring cues to have the same signs, i.e., either both positive or both negative. We can do that by introducing a neighbourhood matrix $\mathbf{V} \in \{0,1\}^{D\times D}$ which determines the connections among cues. Thus, the energy associated with stimuli can be calculated using $-b\mathbf{w}^\top\mathbf{V}\mathbf{w}$, where $b \in \mathbb{R}$. Moreover, we can bias cues towards particular signs, i.e, positive (cues are present) or negative (cues are absent). This is equivalent to adding an extra term $-h\mathbf{w}$, where $h \in \mathbb{R}$.

Finally, we get the energy function in the following form:

$$E(\mathbf{w}) = -h\mathbf{w} - b\mathbf{w}^\top\mathbf{V}\mathbf{w} - c\mathbf{w}^\top\mathbf{x}t. \tag{5}$$

This is a modification of the *Ising model* [4, 12], which has been widely studied in the statistical physics domain. The parameters h and b have certain interpretations in physics,[2] namely, h corresponds to the presence of an external magnetic field, b is the coupling between spins. The last parameter c was not introduced in the original Ising model for magnetic systems and thus has no clear interpretation. However, in our application it determines the correlation between the model and the outcome.

Learning rule Similarly to the Rescorla-Wagner model, in order to obtain the learning rule we want to apply SGD but to the energy function (5). Therefore, we need to calculate the gradient:[3]

$$\nabla_{\mathbf{w}} E(\mathbf{w}) = -h - b(\mathbf{V} + \mathbf{V}^\top)\mathbf{w} - c\mathbf{x}t. \tag{6}$$

Finally, we get the following learning rule:

$$\mathbf{w} := \mathbf{w} + \eta(h + b(\mathbf{V} + \mathbf{V}^\top)\mathbf{w} + c\mathbf{x}t) \tag{7}$$

Let us notice that the learning rule (7) is a generalization of two well-known learning rules. First, for $h = 0$ $b = 0$, and $c = 1$ we get:

$$\mathbf{w} := \mathbf{w} + \eta\mathbf{x}t, \tag{8}$$

which is known as *Hebbian rule* [7].

Second, for $h = 0$, $b = -0.5$, $c = 1$, and for the neighbourhood matrix $\mathbf{V} = \mathbf{I}$, where $\mathbf{I}$ is an identity matrix, we get:[4]

$$\mathbf{w} := \mathbf{w} + \eta(\mathbf{x}t - \mathbf{w}), \tag{9}$$

which is known as *Oja's rule* [16].

[2] Originally, the Ising model has been proposed to investigate properties of idealized magnetic systems. In the Ising model there is a lattice of spins which can take only two values -1 and $+1$. Here, we talk about associative weights which are real-valued.

[3] We use the following property (equation (81) on page 11 in [17]):

$$\frac{\partial \mathbf{x}^\top\mathbf{B}\mathbf{x}}{\partial \mathbf{x}} = (\mathbf{B} + \mathbf{B}^\top)\mathbf{x}.$$

[4] In fact, in order to obtain the original Oja's rule we should write

$$\mathbf{w} := \mathbf{w} + \eta t(\mathbf{x} - \mathbf{w}t)$$

but since $t \in \{-1, 1\}$, and thus $t^2 = 1$, we omit writing t^2 in the final equation.

Combining associative and cognitive approaches In recent studies it has been suggested that associative and cognitive accounts should be combined rather than considered apart [20]. In this paper, we focus on associative learning but here we point out a possible direction of how to incorporate cognitive perspective into the associative one.

We introduce a new concept which we call *surprise factor*, and define it as an absolute difference between the true target and the model's outcome:

$$s = |t - \mathbf{w}^\top \mathbf{x}|. \tag{10}$$

The surprise factor represents the inferential account of a new observation, i.e., the bigger predictive mistake learner makes, the bigger influence should have the observation on the learner. In other words, if the target and the model's outcome are different, the surprise factor reflects the surprise level of the learner about the considered observation.

The proposed quantity can be used to modify the learning rule by changing the learning rate, i.e., $\eta(s) = \eta/s$. In case of observations which are correctly predicted, the learner is not surprised by data and thus makes small contribution to her knowledge. Here we see that the learner has to perform simple cognitive process represented by calculating the surprise factor and then apply the typical associative learning rule with the modified learning rate.

The surprise factor is a simple representation of a cognitive process but we can think of more sophisticated ones. For example, we can try to propose a model of searching in the memory for similar instances and then modify the learning rate. In general, we can formulate an inferential model of the cognitive process which reflects influence of awareness on animal or human perception of the world.

3 Experiments

In the psychology domain there were series of designed experiments which served as benchmark test-cases in evaluating and understanding animal or human learning process [19]. There are two typical experiments which are used in a qualitative comparison of computational models of learning, namely, *backward blocking* [9] and *reduced overshadowing and blocking* [9, 15].

In the following sections the proposed approaches (with parameters: $h = -0.05$, $b = -0.2$, $c = 1$, $\eta = 0.1$, and the neighbourhood matrix $\mathbf{V}$ – an upper triangular matrix of ones), that is,

- without cognitive account (in the experiment called *Our model*);
- with the cognitive account ((in the experiment called *Our model + cognition*),

are compared with the classical learning models (with learning rate equal $\eta = 0.1$):

- the Rescorla-Wagner model;

– Hebbian rule;
– Oja's rule.

The parameters are set arbitrarily. We will not discuss the parameters' influence on the model's performance. We leave this issue for further research.

3.1 Backward blocking

In the backward blocking experiment there are two stimuli, $\mathbf{x} \in \{0,1\}^2$, and one outcome, $t \in \{-1,1\}$. The learning process consists of two training stages (see Table 1). The first phase of training comprises of 10 observations in which both cues occur with the positive outcome. The second stage of training has 10 trials in which only x_1 occurs along with the positive response. The observed phenomenon is that when x_2 is tested by itself at the end of the second stage it evokes lower associative strength than at the end of the first phase.

Table 1. Structure of the learning process for the backward blocking experiment.

Stage	Frequency	x_1	x_2	t
I	10	1	1	1
II	10	1	0	1

The results obtained for the proposed approaches and the three classical learning models are presented in the Figure 1. The associative weights' values are given for learning stage I and II (the learning stage number is denoted in the brackets).

3.2 Reduced overshadowing and blocking

In the reduced overshadowing with blocking experiment there are four stimuli, $\mathbf{x} \in \{0,1\}^4$, and one outcome, $t \in \{-1,1\}$. The learning process consists of two training stages (see Table 2). The first phase of training comprises of 6 observations in which only x_1 occurs with the positive outcome and then 6 observations in which x_3 occurs with the negative outcome. The second stage of training has 6 observations in which x_1 and x_2 occur along with the positive response and next 6 observations in which x_3 and x_4 occur with the positive response. The observed phenomenon is that first x_3 entails no outcome and thus it reduces overshadowing when x_3 occurs together with x_4. Additionally, at the end of the learning process the associative strength of x_2 should be less than x_4 because the x_2 is blocked by x_1.

The results obtained for the proposed approaches and the three classical learning models are presented in the Figure 2. The associative weights' values are given for learning stage Ia, Ib and IIa, IIb (the learning stage number is denoted in the brackets).

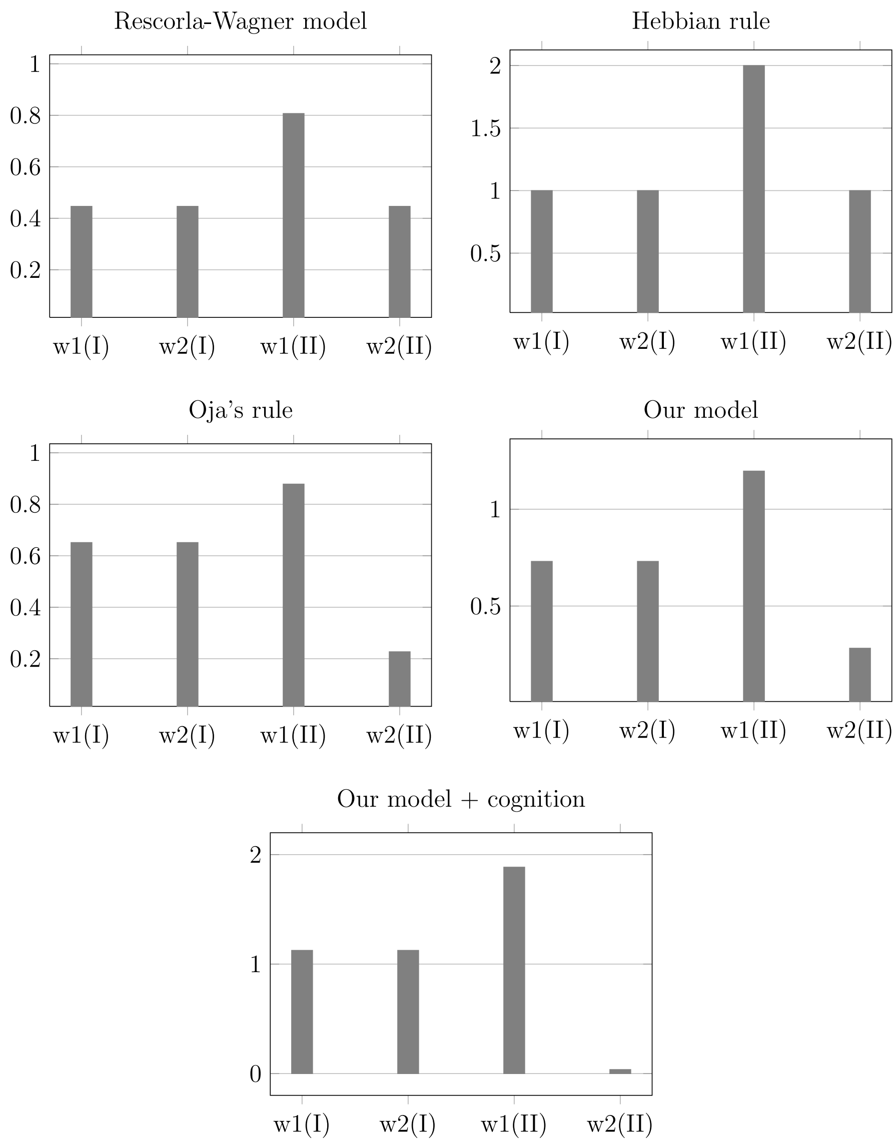

Fig. 1. Summary performance comparison of all considered in the paper learning rules in the backward blocking experiment.

Table 2. Structure of the learning process for the reduced overshadowing and blocking experiment.

Stage	Frequency	x_1	x_2	x_3	x_4	t
Ia	6	1	0	0	0	1
Ib	6	0	0	1	0	-1
IIa	6	1	1	0	0	1
IIb	6	0	0	1	1	1

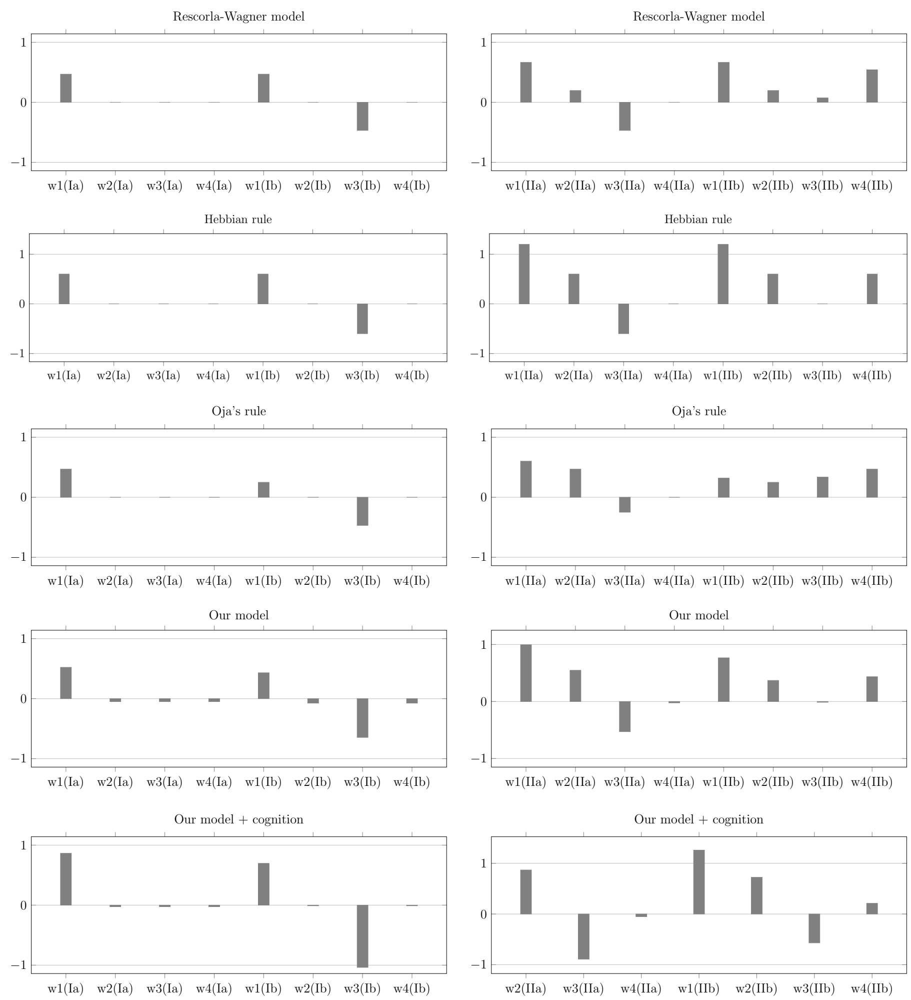

Fig. 2. Summary performance comparison of all considered in the paper learning rules in the reduced overshadowing and blocking experiment.

3.3 Discussion

In the first experiment, the obtained results (see Figure 1) show that the Rescorla-Wagner model does not exhibit backward blocking which is a known fact [14]. Similarly, the Hebbian rule simply leads to accumulating the number of co-occurrences of cues and thus it is insensitive to the backward blocking phenomenon. On the other hand, the Oja's rule and both of our proposed models are able to model the backward blocking. This result indicates the importance of introducing $\mathbf{w}^\top \mathbf{V}\mathbf{w}$. In fact, this part of the energy function plays a role of a *regularizer* if we notice that $\mathbf{w}^\top \mathbf{V}\mathbf{w} = \|\mathbf{w}\|^2_{\mathbf{V}}$. [5] In other words, it tries to *pull* all weights' values towards zeros and therefore the Oja's rule and our approaches exhibit backward blocking.

In the second experiment, the obtained results (see Figure 2) are quite surprising. The Rescorla-Wagner model exhibit both reduced overshadowing and blocking. However, our proposition without and with cognition shows more evidently these both phenomena. It is important that our model indicates strong associative weights for cues 1, 2, and 4, and zero or strong negative (for our proposition with cognition) association for cue 3. At the end, let us notice that Hebbian rule failed completely. The Oja's rule performed better but it put too much weight on last observations and thus the cue 3 and cue 4 have too strong associative strengths.

4 Conclusions

In this paper, the new computational model of associative learning was proposed. It is based on the Ising model which has been successfully applied in the statistical physics [12] and other domains [3, 10, 21]. It was shown that the obtained new learning rule (7) generalizes the Hebbian rule (8) and the Oja's rule (9). Next, the fashion of incorporating the cognitive account into the obtained associative learning rule is proposed. Possibly, this indicates a new direction for future research. At the end of the paper, experiments were carried out for testing the backward blocking and the reduced overshadowing and blocking phenomena. The obtained results have revealed the supremacy of our model over the classical approaches to the associative learning.

In the ongoing research we develop the probabilistic approach to the presented problem, i.e., application of Boltzmann machine [1]. In this paper, we have pointed out the possibility of combining associative and cognitive accounts. Considering the experimental results this idea seems to be an interesting direction for further investigations. Additionally, we have assumed an arbitrary values of parameters of the model. The parameters' values should be fitted to individuals basing on real data. However, we leave investigating this aspect as future research.

[5] Since $\mathbf{V}$ is an upper triangular matrix of ones, its all eigenvalues are positive and hence it is a positive definite matrix.

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